Labor Market Recoveries Across the Wealth Distribution

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Abstract

I study how wealth impacts workers' job-switching behavior and their earnings through a *precautionary job-keeping motive*. All else equal, low-wealth workers are less willing to switch jobs because such moves increase their short-term risk of job loss. I quantify this channel using a search and matching model where wages are determined by a generalized alternating offer bargaining protocol accommodating risk-aversion, wealth accumulation, and on-the-job search. Precautionary job-keeping accounts for half the earnings gap between low- and high-wealth workers after the Great Recession. The Pandemic stimulus weakened this motive leading to the strong job-switching recovery the US has recently experienced.

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1 Introduction

Different groups of workers display different labor market outcomes over the business cycle. While several dimensions of worker heterogeneity have been explored, including income (e.g. Heathcote, Perri and Violante 2020), sex, age, race, and education (e.g. Elsby, Hobijn and Şahin 2010), little has been said about the labor market experience of workers with different wealth. Wealth, however, is a natural variable to consider because it proxies for workers' ability to smooth consumption in the face of adverse shocks.

Low-wealth workers experience more pronounced labor market downturns than high-wealth workers. After the Great Recession, for instance, workers with below median wealth experienced on average a ten percent decline in real earnings, a fall that took more than four years to recover. In contrast, workers with above median wealth experienced only a small and short-lasting drop in earnings. Because workers use their wealth to smooth consumption, the adversity low-wealth workers experience is twofold: they not only endure the worst consequences of recessions but they are also worst prepared to confront them. This pattern holds even after accounting for standard worker characteristics, suggesting wealth itself plays a unique role in explaining it.

In this paper I document a relationship between wealth and workers' job-switching behavior. I argue that differences in job-switching across the wealth distribution contribute to the deeper fall in earnings experienced by low-wealth workers following recessions. To do so, I build a quantitative search and matching model with incomplete markets and on-the-job search that generates the observed patterns in labor market flows in equilibrium. This model cannot be solved using traditional wage setting protocols because, when incorporating on-the-job search, these either assume linear utility or hand-to-mouth consumption, assumptions incompatible with the questions this paper tackles. To overcome this challenge, I develop a *generalized* alternating offer bargaining (AOB) protocol, building on Hall and Milgrom (2008), which accommodates risk-aversion, asset accumulation, and on-the-job search. Additionally, the model includes a salient feature of the data that I document empirically: workers who switch jobs experience a persistent increase in their risk of subsequent job-loss.

The combination of on-the-job search, incomplete markets, and risky job-switching gives rise to two forces which enable the model to (i) explain the cyclical differences in job-switching across the wealth distribution, (ii) explain over half of the observed earnings gap between high- and low-wealth workers following the Great Recession, and (iii) rationalize the Great Reallocation that affected the US economy in the post-Pandemic period through the generous fiscal support received by households.

The model gives rise to a *precautionary job-keeping motive* which, all else equal, leads low-wealth workers to be less willing to switch jobs just so they can avoid the additional risk of job-loss that switching entails. Because job-switches are associated with earnings increases, this motive hinders the earnings recovery of low-wealth workers. There is a second phenomenon that the model's dynamic selection forces give rise to. I denote this the *tenure-wealth correlation*, which leads low-wealth workers to be more exposed to job-loss because of the lower tenure jobs they tend to occupy. This implies that low-wealth workers experience recoveries that are interrupted by more frequent unemployment spells, in turn depressing their earnings growth relative to that of high-wealth workers.

I start by studying the relationship between wealth and labor market flows in the Survey of Income and Programs Participation (SIPP). I find that the standard deviation of the cyclical component of the job-switching probability for workers in the bottom half of the wealth distribution is twice that for workers in the top half. That is, after recessions, the rate at which workers switch jobs falls by more for low-wealth workers than for high-wealth workers. Additionally, I find that a similar pattern holds for the job-losing rate, namely low-wealth workers lose their jobs at a higher rate than high-wealth workers once a recession hits the economy.

I next develop a model that can speak to these empirical findings. The model integrates an incomplete markets, heterogeneous agent framework into a search and matching model à la Diamond (1982) and Mortensen and Pissarides (1994) (DMP) with on-the-job search. The model includes three key ingredients: (i) risk-averse workers, (ii) asset accumulation, and (iii) risky job moves.

The first two ingredients are standard in macroeconomics. However, for technical convenience, they are seldom included by the search and matching literature. Most papers that include these, such as Krusell, Mukoyama and Şahin (2010), do not incorporate on-the-job search. I devise a *generalized* AOB protocol to determine the wages firms and workers agree on. This wage setting scheme is micro-founded, parsimonious, and can be easily applied to a large class of models.

On-the-job search requires three parties to be involved in wage negotiations: the worker, the incumbent firm, and the poaching firm. To incorporate this three-party negotiation, I extend the standard AOB protocol between one firm and one worker to an environment where two firms compete to attract one worker. This *generalized* AOB framework goes beyond that in Cahuc, Postel-Vinay and Robin (2006) by allowing for risk-aversion with no restrictions to the state space. I prove that its solution boils down to three cases. If the incumbent is significantly more productive than the poacher, it retains the worker at the same wage. If the two firms are similarly productive, they Bertrand-compete for the worker. If the poacher is significantly more productive, it negotiates one-on-one with the worker as in the standard AOB protocol.

The third key ingredient of the model, risky job moves, asserts that workers who switch jobs face a persistent increase in probability of job loss. It is well documented (e.g. Martellini, Menzio and Visschers 2021) that the probability of job-loss decreases in tenure. I show that a similar pattern holds for job-switchers who, following a job-switch, experience an increase in their job-loss probability. To quantify the additional risk of job-loss that job-movers face I estimate an event study using the SIPP. I find that the increase in job-loss probability due to job-switching is 7.4 percentage points in the first fifteen months at a new job. This estimate is large given that the typical US worker has, over the same fifteen month period, an 18 percent chance of being laid-off.

This empirical pattern emerges endogenously in the model because, in the spirit of Jovanovic (1979), when a worker and firm first meet, the idiosyncratic quality of their match is unknown and is only learned with time. In the initial periods of the match, workers with low-quality matches are revealed and laid off while only workers with high-quality matches are retained delivering a probability of job-loss that declines with tenure.

Model Mechanisms. The precautionary job-keeping motive delivers the empirical variation in the job-switching probability across the wealth distribution. This is a causal mechanism that makes low-wealth workers more conservative in their job-switching decisions because they are less willing to bear the risk that switching jobs entails. While high-wealth workers can rely on their assets to smooth consumption during unemployment, this is not an option for workers with low wealth, who will forgo the increase in earnings associated with switching jobs just so they can avoid additional risk of falling into unemployment.

I use cross-sectional evidence from SIPP to test this. As the model predicts, I find evidence that workers with higher wealth-to-income ratios are more likely to switch jobs. Moreover, this effect is highly non-linear: the same increase in wealth has a larger (positive) effect on the propensity to switch jobs at the bottom of the wealth distribution than at the top. These reduced form results are consistent with the findings from a recent experiment in which some individuals in Stockton, California randomly received monthly payments for $500. West et al. (2021) find that "guaranteed income enabled shifts in employment by giving recipients the emotional and financial capacity for risk taking."

Because recessions lead to loss of wealth, they make workers more sensitive to risk and in turn exacerbate precautionary job-keeping which depresses overall job-switching. However, the fall in the job-switching probability is larger for low-wealth workers because, with concave utility, the same drop in wealth results in a larger increase in their marginal utility compared to high-wealth workers, leading them to become effectively more risk averse. This reasoning explains why the cyclical component of job-switching is more volatile at the bottom of the wealth distribution.

The model gives rise to a second force, the tenure-wealth correlation, which helps explain the empirical variation in the job-losing probability across the wealth distribution. Unlike precautionary job-keeping, which underscores a *causal* relationship, the tenure-wealth correlation results from the model's dynamic selection forces. It reflects the tendency of low-wealth workers to occupy low tenure jobs. This occurs because some workers are unlucky and experience long or frequent streaks of unemployment. These unlucky workers tend to have low tenure and low wealth. They have low-tenure because they recently exited unemployment and started new jobs; they have low wealth because they depleted their savings to smooth consumption while unemployed. Hence low-wealth workers tend to be in low-tenure jobs with a higher probability of job-loss. This force is exacerbated during recessions because the pool of unemployed workers grows. As these workers re-enter the labor market they take up new, low-tenure positions. This results in a large mass of workers, who are low-wealth because they recently experienced unemployment, finding themselves in low-tenure jobs that are more likely to lead to job loss. In other words, in recessions the job-loss probability of low-wealth workers increases by more than for high-wealth workers. This force leads low-wealth workers to endure slower earnings recovery because they experience more frequent unemployment spells.

Main results. The model exactly matches differences in the cyclical behavior of the job-switching probability across the wealth distribution and can also account for some of the distributional variation in the job-losing probability. I show these results rely crucially on the presence of job-switching risk by comparing the benchmark model to a *naïve* version of the model in which the job-loss probability is constant rather than decreasing in tenure. A constant job-loss probability eliminates not only the precautionary job-keeping motive, since job-switchers no longer face a higher probability of job-loss, but also eliminates the tenure-wealth correlation, since tenure becomes a meaningless concept.

The model helps explain why, after the onset of recessions, earnings fall more for low-wealth workers. Because of precautionary job-keeping, low-wealth workers become more hesitant to switch to new jobs following recessions. While this spares them additional risk of job-loss, it also precludes them from accepting better, higher-paying jobs. Because of the tenure-wealth correlation, workers with low wealth who tend to be in low-tenure jobs are more exposed to unemployment spells which limit their participation in the labor market and prevent them from climbing up the job ladder. These two phenomena explain half of the gap in the earnings recovery experienced by low-wealth workers relative to high-wealth ones following the Great Recession.

Finally, I apply the model to study the Pandemic Recession and argue that the generous fiscal stimulus provided by the US government sustained the recovery of job-switching over this period, a phenomenon people have characterized as the "Great Reallocation." According to the model, the injection of wealth onto workers' balance sheets alleviated their precautionary job-keeping motive, incentivizing them to switch jobs. I show that, under a counterfactual scenario in which the government did not provide fiscal stimulus, the Great Reallocation would not have occurred. Absent the stimulus, the (quarterly) job-switching probability would have dropped by an extra twenty basis points at its trough.

Related literature. This paper contributes to the search and matching literature by considering a new environment with on-the-job search and incomplete markets in which wages are endogenously determined via a *generalized* AOB protocol and where workers are both risk-averse and accumulate assets. Compared to Krusell, Mukoyama and Şahin (2010), pioneers in embedding incomplete markets in a search and matching model, I allow workers to switch jobs. While Lise (2013) also includes job-switching it does not endogenize wage offers. Recent work has studied on-the-job search in richer environments. Fukui (2020) includes risk-averse workers in a wage-posting setting. Unlike *generalized* AOB, wage-posting misses a feature that is pervasive in US data: it does not allow for renegotiations. According to the NY Fed Survey of Consumer Expectations, roughly half of workers who receive outside offers try to re-negotiate their wages with the incumbent firm. Moscarini and Postel-Vinay (2022) include both risk-aversion and asset accumulation by having firms Bertrand-compete for workers. Unlike *generalized* AOB, this solution suffers from two limitations. First, it does not allow for surplus-sharing between agents, effectively endowing firms with all the bargaining power. Second, it leads to implausibly small wage gains when workers move from very unproductive to very productive jobs.

My model nests Cahuc, Postel-Vinay and Robin (2006) by relaxing their assumptions of linear utility and hand-to-mouth consumption. The first of these gives rise to a simple surplus-splitting rule, the second allows to ignore workers' consumption-savings problem and in turn the complications arising from having wealth as a state variable. While technically convenient, these assumptions clash with the questions this paper tackles.

This paper advances our understanding of the heterogeneous labor market outcomes workers experience. While Krusell et al. (2017) use a search model to match the cyclical properties of aggregate labor market flows, I show that matching these moments across the wealth distribution is important to explain heterogeneous earnings dynamics. In doing so, I complement the large literature that has studied the heterogeneous effects of recessions. Some of these papers look at workers differing by income (e.g. Heathcote, Perri and Violante 2020, Kramer 2022), or by demographic characteristics such as race, sex, age, and education (e.g. Elsby, Hobijn and Şahin 2010). However, as Hall and Kudlyak (2019) and Gregory, Menzio and Wiczer (2021) find, large differences in labor flows across workers remain even after accounting for these demographic traits. Unlike the existing work, my paper looks at wealth as the source of heterogeneity among workers. Wealth is a natural dimension to look at because it proxies for workers' ability to smooth consumption in adverse times and, as such, it is informative of how well workers fare during recessions. The model I develop brings new forces tying wealth to workers' job-switching and job-losing behavior that are quantitatively important to explain the earnings gap experienced across the wealth distribution during and after recessions.

Finally, this paper contributes to a growing literature tying labor market decisions to wealth. Much of this work has concentrated on the labor market decisions of unemployed individuals. Krusell, Mukoyama and Şahin (2010) study the channels through which unemployment benefits affect worker welfare. Eeckhout and Sepahsalari (2021) and Huang and Qiu (2022) study how wealth affects the jobs unemployed workers choose to apply for. My work points out that wealth has a larger role to play: it affects not only the employment decisions of the unemployed but also the job-switching decisions of those already employed. This paper is the first to study the macroeconomic effects of wealth through its role on workers' job-switching behavior.

2 Background: Labor Market Outcomes and Wealth

A striking feature of the recovery from the Great Recession is how unequal it was. Low-wealth workers suffered worse outcomes than high-wealth workers did. While this behavior is true for a variety of labor market indicators it is best summarized by earnings. Figure 1 shows the evolution of real labor earnings for low (red) and high-wealth (blue) workers around the 2001 and the 2007-2009 recessions. Labor earnings are defined as real gross wages paid by the worker's main employer. These series exclude the unemployed, the self-employed,[1] those outside the labor force, as well as part-time workers working fewer than 35 hours per week. I adopt this definition of labor earnings because it better captures the *quality* margin of employment, the primary concern of this paper.

[1] Risk factors other than job-loss (e.g. credit risk) are likely to be more relevant for the self-employed.

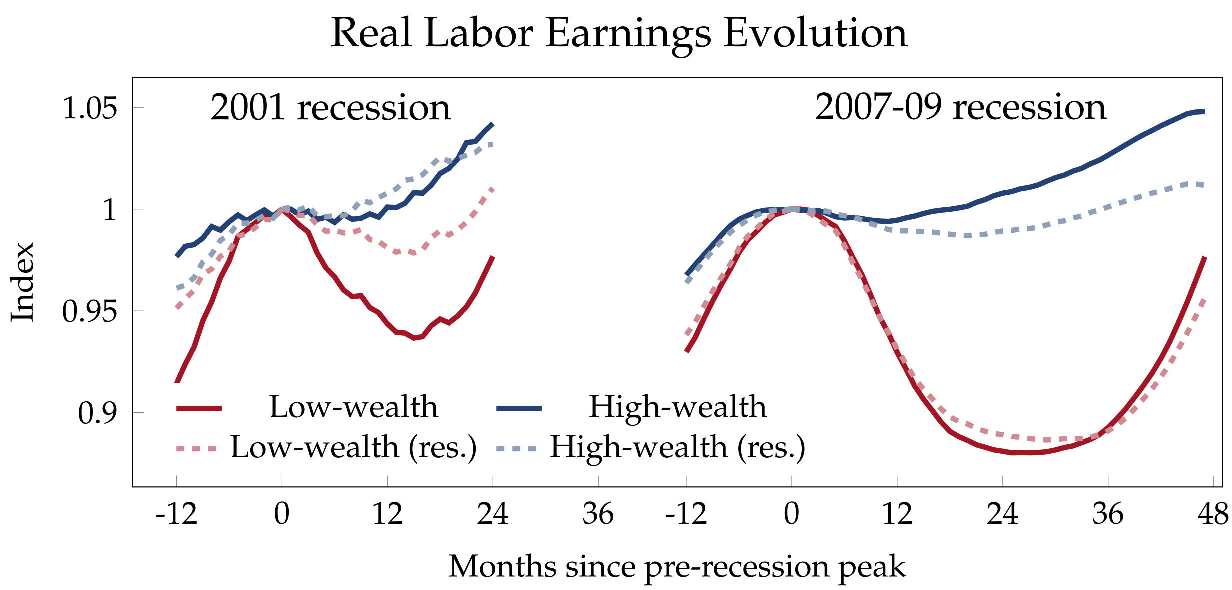


Figure 1: Labor income evolution around recessions, indexed at pre-recession peak. Top half (high-wealth) and bottom half (low-wealth) of net worth distribution. Analysis for raw data (solid) and data residualized by a polynomial in age, sex, race, tenure, work type (union, private, govt.), education and industry fixed effects. Dates missing due to SIPP gaps between surveys are interpolated. The displayed series is smoothed using a 12-month moving average. Soure SIPP and own calculations.

Wealth is measured as net-worth following Kaplan and Violante (2014). The threshold separating low- and high-wealth workers is median wealth. Here and in what follows, I dynamically sort workers at each period across the median threshold. I do so for two reasons: first because the panel dimension is too short to cover the '07-'09 recession and its recovery and second because SIPP has gaps between surveys leading to no observations for some of the dates displayed.[2] For each recession and each group, earnings are normalized to their pre-recession peaks.

The picture painted is striking: following the Great Recession, earnings for workers in the bottom half of the wealth distribution fell by more than 10% and took more than four years to recover to 2007 levels, earnings for workers in the top half of the wealth distribution experienced only a minor, short-lasting decrease. Though less extreme, a similar picture can be painted for the 2001 recession. In addition, these patterns hold true when residualizing by standard controls[3] (dashed lines), indicating much of this empirical earnings gap remains unexplained.

Low-wealth workers are generally worst equipped to confront downturns because

[2] I choose to interpolate the aggregate time series shown in the figure rather than extrapolating individual earning dynamics and then aggregating.

[3] These are gender, race, industry of occupation, education, and a polynomial in age.

they cannot rely on their savings to get by in case they are hit by adverse shocks. In addition, figure 1 shows, low-wealth workers suffer larger falls in their earnings. This means that those workers who are worst equipped to confront recessions also suffer the worst consequences from them. This is why it is so important to understand what drives the heterogeneity in the labor market recoveries of workers with different wealth.

To understand where these earnings differences come from, it is natural to look at the labor flows of these workers – labor earnings are after all determined by the jobs workers hold. Table 1 displays the standard deviation and persistence of the cyclical component of the job-finding (UE), job-losing (EU), and job-switching (EE) probabilities at the quarterly frequency. The take-away from this table is that the behavior displayed by labor earnings is not unique to it: the cyclical components of the job-switching and the job-losing rates are also more volatile for workers in the bottom half of the wealth distribution. This means that, after a recession, the rate at which low-wealth workers *switch* jobs falls by more and takes longer to recover relative to the rate of high-wealth workers; the rate at which low-wealth workers *lose* their jobs increases by more than for high-wealth workers. In contrast, the cyclical component of the job-finding rate (UE) displays no significant difference across wealth.

	Mean (%)			Stdv.			Persistence		
	all	low-wealth	high-wealth	all	low-wealth	high-wealth	all	low-wealth	high-wealth
UE	55.70	51.23	61.70	5.44 (0.847)	5.01 (0.762)	6.07 (0.944)	0.847 (0.762)	0.9634 (0.037)	0.9617 (0.041)
EU	2.81	3.92	2.14	1.20 (0.177)	1.55 (0.165)	0.91 (0.136)	0.8914 (0.073)	0.8894 (0.065)	0.8888 (0.073)
EE	4.14	5.37	3.34	1.19 (0.275)	1.54 (0.344)	0.99 (0.237)	0.9109 (0.088)	0.9104 (0.087)	0.9042 (0.085)

Table 1: Quarterly labor market flow rates across the distribution of net worth excluding housing. "All" is entire sample, "low wealth" and "high wealth" are the bottom and top halves of the net worth ex. housing distribution. Standard deviations and persistence parameters are computed on the Hamilton-filtered rates. Persistence is the AR(1) coefficient. Bootstrapped standard errors following Politis and Romano (1994) are shown in parenthesis. All data are computed using SIPP 1996-2013.

These differences across wealth persist even when residualizing the data by standard controls.[4] In the next section I develop a model that can make sense of the heterogeneity in these labor market flows and in turn can speak to the earnings gaps across the wealth

[4]In Appendix A.1 I show the moments residualized by standard worker characteristics. Gregory, Menzio and Wiczer (2021) also show that large differences in labor flows across workers persist after accounting for standard controls. Low-wealth workers in my model display a similar job-losing behavior as the workers they denote as "gamma" types, indicating that wealth may explain some of what lies behind the statistical classification of workers they document.

distribution observed in recent recessions.

3 Model

This model incorporates incomplete markets in a search and matching model with random search. There are four agents in this model: households that are either employed or unemployed; firms that are either in search of a worker or actively producing goods; capitalists that rent capital to firms; and the government that taxes households to pay for unemployment benefits and government transfers. I go over each of these in detail.

3.1 Households

Households can either be unemployed or employed. If employed, they work for a firm of type $n \in \{1, \ldots N\}$ where n indexes the labor market the firm belongs to. All firms in labor market n have productivity p_n that is increasing in n.

Unemployed. Unemployed agents choose how much to consume, c, and save, a', using their gross wealth, $(1+r)a$, unemployment benefits, b, and a lump sum government transfer, T. Unemployed agents are always in search of a job. They randomly get a chance to search in one out of N possible labor markets. Specifically, they search in labor market n according to the c.d.f. $G(n|0)$ with probability mass function $g(n|0)$.[5] If searching in labor market n, the agents find a job with endogenous probability λ_n, otherwise they remain unemployed. The problem they solve is

$$\begin{aligned} U(a,z) &= \max_{c,a'} u(c) + \beta \mathbb{E}\left[\left(1 - \sum_{n=1}^{N} g(n|0)\lambda_n\right) U(a',z') + \sum_{n=1}^{N} g(n|0)\lambda_n E^u(a',z',n)\right] \quad (1) \\ \text{s.t.} &\quad c + a' = (1+r)a + b + T \qquad \text{and} \qquad a' \geq \underline{a} \end{aligned}$$

In addition to wealth, all agents have an idiosyncratic productivity z that evolves according to a first order Markov process, $z' \sim F(z'|z)$. This idiosyncratic term affects how productive agents are when engaged with a firm. Because the process $F(\cdot)$ is persistent, z affects the value of unemployed workers not contemporaneously but through the continuation value. When an unemployed agent meets a firm, the value from the match is

[5] $g(\cdot)$ delivers computational parsimony because whenever $g(n|0) = 0$ it is unnecessary to compute wages for unemployed workers searching in labor market n. A similar logic will later hold for workers switching between n and n' if $g(n'|n) = 0$.

$E^u(\cdot)$, which can be rewritten as

$$E^u\left(a',z',n\right) \equiv E\left(a',z',w^U\left(a',z',n\right),n,0\right) \tag{2}$$

where 0 indicates the worker starts with no tenure at the new job, n indicates the labor market the agent finds employment in, and $w^U(a',z',n)$ is the wage the firm and worker agree on. This wage, which will be discussed in detail in the following section, depends on the state variables (a',z',n) because so do the worker's and firm's outside options.

Employed. Employed agents engaged with a firm on rung n, earn after-tax income $(1-\tau)w$, where the wage w is pre-established, and have tenure j at their job.

A crucial ingredient of the model that I later validate in the data is that workers' probability of job loss is decreasing in tenure. That is, the longer a worker is at a firm, the less likely they are to be laid off. Thus, the job-loss probability, denoted $\sigma(j)$, is such that $\sigma(j) \geq \sigma(j+1)$. While I provide a microfoundation for this declining hazard rate at the end of this subsection, assume for now that workers of tenure j separate into unemployment with *exogenous* probability $\sigma(j)$. If they do not fall into unemployment, they either continue the relationship with the current firm or get an offer from a new firm on a different rung. Only a random share s of workers on rung n is allowed to search for a new job in any given period. If searching, the probability of searching on rung n' is $g(n'|n)$. Conditional on being able to search on rung n', the probability of an offer from a firm is $\lambda_{n'}$. If the worker does not get an offer, they stay in their current job, earning the same wage but gaining a period in tenure. If the worker does get an offer, they must decide whether to move to the new firm or stay with the old one. In either case the worker negotiates a new wage contract. The problem the worker faces is

$$\begin{aligned} E\left(a,z,w,n,j\right) &= \max_{c,a'} u\left(c\right) + \beta \mathbb{E}\Bigg\{\sigma\left(j\right)U\left(a',z'\right) \\ &\qquad + \left(1-\sigma\left(j\right)\right)\Bigg[\left(1 - s\sum_{n'=1}^{N} g(n'|n)\lambda_{n'}\right)E\left(a',z',w,n,j+1\right) \\ &\qquad + s\sum_{n'=1}^{N} g(n'|n)\lambda_{n'} E^e\left(a',z',n,n',j\right)\Bigg]\Bigg\} \\ \text{s.t.} \qquad & c + a' = Ra + (1-\tau)w + T \end{aligned} \tag{3}$$

where the term $E^e(\cdot)$ represents the worker's value in case they get an offer from a firm in rung n'. This term can be rewritten as

$$E^e\left(a',z',n,n',j\right) \equiv \max_{\phi\in\{0,1\}} (1-\phi)\left\{E\left(a',z',w_E^{\text{stay}}\left(a',z',n,n',j\right),n,j+1\right)+\eta^{\text{stay}}\right\} \\ +\phi\left\{E\left(a',z',w_E^{\text{switch}}\left(a',z',n,n',j\right),n',0\right)+\eta^{\text{switch}}\right\} \tag{4}$$

where the worker can choose to stay ($\phi = 0$) or switch ($\phi = 1$) to firm in labor market n'. Furthermore, when making this decision workers are subject to i.i.d. extreme value taste shocks $\eta^{\text{stay}}, \eta^{\text{switch}} \sim \mathcal{EV}\left(\alpha^{EV}\right)$.

Consider the tradeoff workers face when switching jobs. There is never a benefit[6] to switching to lower productivity firms. The benefit from switching to a higher productivity firm $n' > n$ is clear: because the firm is more productive, it can offer a higher wage, that is $w_E^{\text{switch}}\left(a',z',n,n',0\right) > w_E^{\text{stay}}\left(a',z',n,n',j\right)$. However, workers who switch to a new firm give up their tenure: by staying at the incumbent firm workers gain a period in tenure, going from j to $j+1$, by switching to the poaching firm, workers' tenure falls to 0. The cost of this comes in the form of increased probability of job loss in later periods. Thus, the trade-off job-switchers face is between a higher wage and a less stable job more likely to lead to unemployment. This trade-off is key to the results in the paper.

Microfounding $\boldsymbol{\sigma(j)}$. The seminal work by Jovanovic (1979) provides a simple microfoundation for the downward-sloping $\sigma(j)$. This work theorizes that workers and firms slowly learn about the quality of their match. When a worker and firm first sign a contract, they do so with limited information. As time goes by the firm learns how good the match with the worker really is and decides whether to keep or lay off the worker.

Assume that in the first $J-1$ periods of a match the worker is in "training" and is supervised by the firm. The firm observes the worker and forms beliefs about their potential. Worker potential is idiosyncratic to the match and it is high (H) with probability π^H and low (L) with probability $1-\pi^H$. Once the training stops, at J, the worker continues to produce at full capacity if they are high potential but produces no output if they are low potential. In the initial $J-2$ periods the firm only gets a noisy signal of the unobserved worker potential. It uses this signal to determine whether to keep or lay off the worker. At $J-1$, the true potential of the worker is revealed and the remaining L workers are laid off.

At $j \geq J$ all workers have a common job-loss probability σ. In the initial periods $j = 1, \ldots J-2$, the firm receives one of two possible signals about worker potential. It

[6] Other than that provided by the taste shocks which, for sake of argument, I ignore here.

either spots the worker committing a mistake and thus revealing they have low potential, in which case the firm fires the worker, or it observes no mistake and the worker is laid off with probability σ. The probability a low potential worker actually commits a mistake is α^L. This leads to layoff probabilities $\sigma(j)$ which is decreasing in j.[7]

$$\sigma(j) = \begin{cases} (1-\pi^H)\cdot(1-\alpha^L)^J\cdot\alpha^L+\sigma & \text{if } j<J \\ \sigma & \text{if } j\geq J \end{cases} \tag{5}$$

3.2 Firms

Firms can either be vacant or active. Vacant firms are in search of one worker. Active firms engage in production with one worker. Each labor market n is distinguished by its own mass of (identical) vacant and active firms all of which have productivity p_n increasing in n.

Active Firms. An active firm on rung n is paired with worker of type (a, z, w, n, j) where w is the wage the two parties negotiated either at the start of the match or the last time the worker had an outside offer. Firms on rung n paired to workers with idiosyncratic productivity z produce according to the constant return to scale technology

$$y_n = F(k_{-1}, L) = Zk_{-1}^{\alpha}L^{1-\alpha} \qquad \text{s.t. } L = p_n\cdot z$$

where L are the effective units of labor from the match, Z is aggregate productivity, and k_{-1} is the capital the firm uses in production.

In any given period there is a probability $\sigma(j)$ the match ends. If the match continues, with probability $s\cdot\sum_{n'=1}^{N}g(n'|n)\lambda_{n'}$ the worker receives an outside offer and with the complement probability the firm and worker continue the existing contract. The value to the firm is

$$J(a,z,w,n,j) = \underbrace{y_n - r^K k_{-1} - w}_{\text{flow profits}} \tag{6}$$

$$+\frac{1}{1+r}\mathbb{E}\left\{\sigma(j)\underbrace{V(n)}_{\text{match ends}} + (1-\sigma(j))\left[s\sum_{n'=1}^{N}g(n'|n)\lambda_{n'}\underbrace{J^{ee}(a',z',n,n',j)}_{\text{outside offer}}\right.\right.$$

[7] An alternative formulation, closer to Jovanovic (1979), would have output directly and contemporaneously affected by worker type and in turn would require the firm to evaluate whether to keep the worker. This would lead to a similar result but would require the addition of at least one state variable to keep track of fluctuating worker output.

$$+ \left(1 - s \sum_{n'=1}^{N} g(n'|n)\lambda_{n'}\right) \underbrace{J\left(a', z', w, n, j+1\right)}_{\text{no outside offer}}\Bigg]\Bigg\}$$

where J^{ee} is the value of the firm on rung n in case its worker is offered a job on rung n'. This value can be rewritten as

$$J^{ee}(\cdot) = \begin{cases} V(n), & \text{if worker switches} \\ J\left(a', z', w_E^{\text{stay}}(a', z', n, n', j), n, j+1\right), & \text{if worker stays} \end{cases} \tag{7}$$

If the worker switches, firm n opens a vacancy with value $V(n)$, if the worker stays, they renegotiate a wage $w_E^{\text{stay}}(a', z', n, n', j)$ with the firm.

Vacant Firms. On each rung n there are vacant firms that pay a fixed cost $\kappa \cdot p_n$ to post a vacancy. Next period they meet a worker with probability q_n, otherwise they remain vacant. The problem they face is

$$V(n) = -\kappa p_n + \frac{1}{1+r}\left[(1-q_n) V(n) + q_n J_0(n)\right] \tag{8}$$

where $J_0(n)$ is the expected value of a newly active firm on rung n

$$\begin{aligned} J_0(n) &= \int_{x^u} g(n|0)\, J^0\left(x^u, w^u(x^u; n)\right) d\Psi^u(x^u) \\ &+ \int_{x^e} s \sum_{n'>0} g(n|n')\Big[\underbrace{\varphi(x^e, n')}_{\text{pr. of poaching}} J^0\left(x^e, w^e_{\text{switch}}(x^e, n')\right) + (1-\varphi(x^e, n'))V(n)\Big]\, d\Psi^e(x^e) \end{aligned}$$

where $x^u \equiv (a, z)$, $x^e \equiv (a, z, n, j)$ and $\Psi^u(x^u), \Psi^e(x^e)$ are distributions over x^u, x^e. $J^0(\cdot)$ is the same as $J(\cdot)$ defined in equation (6) but without allowing the worker to switch in the very first period of the match.

$J_0(n)$ is a weighted average of the value of the firm upon meeting unemployed workers, x^u, and employed workers, x^e. The model calibration will imply unemployed workers always accept the jobs they are offered. However, when a vacant firm meets a worker in labor market n' it only poaches them successfully with probability $\varphi(a, z, n', n, j)$.[8] In this case they negotiate a wage w^e_{switch} and start actively producing, otherwise they do not poach the worker and remain vacant.

[8] This poaching probability is derived from the job-switching problem in equation (4).

Profits. Aggregate profits Π are the sum of flow profits net of vacancy costs from all firms, that is

$$\Pi = \sum_{n=1}^{N} \left[\int_{x^e(n)} \left(y_n - r^K k_{-1} \left(x^e \left(n \right) \right) - w(x^e(n)) \right) \, d\Psi^e(n) - \kappa v_n p_n \right] \tag{9}$$

where $x^e(n) \equiv (a, z, w, j; n)$ and the last addend are the vacancy-filling costs in labor market n.

3.3 Capitalist and Government

There are two more agents in this economy: the capitalist and the government. The capitalist rents out capital to the firms, the government transfers resources from some agents to others making sure its budget is always balanced.

Capitalist. There is a representative capitalist who rents out capital to firms. The capitalist chooses how much capital to bring over into the next period. Her objective is to maximize the discounted stream of dividends by choosing the total amount of capital to bring into the following period. This capital investment decision is subject to quadratic adjustment costs. The problem the capitalist faces is

$$(1 + r_t) \mathcal{P} \left(K_{-1} \right) = \max_{K} D_t + \mathcal{P} \left(K \right) \tag{10}$$

$$\text{s.t. } D_t = r^K K_{-1} - \left[K - (1 - \delta) K_{-1} + \frac{1}{2\delta\epsilon_I} \left(\frac{K - K_{-1}}{K_{-1}} \right)^2 K_{-1} \right] \tag{11}$$

where investment $K - (1 - \delta) K_{-1}$ is subject to adjustment costs $\frac{1}{2\delta\epsilon_I} \left(\frac{K-K_{-1}}{K_{-1}} \right)^2 K_{-1}$. From this, the usual Q-theory equations follow for Tobin's Q and its law of motion

$$Q \;:=\; \mathcal{P}' \left(K \right) = 1 + \frac{1}{\delta\epsilon_I} \left(\frac{K - K_{-1}}{K_{-1}} \right) \tag{12}$$

$$Q_{-1} \;=\; \frac{1}{1+r} \mathbb{E} \left[r^K - \frac{K}{K_{-1}} + (1 - \delta) - \frac{1}{2\delta\epsilon_I} \left(\frac{K}{K_{-1}} - 1 \right)^2 + \frac{K}{K_{-1}} Q \right] \tag{13}$$

Government. The government has one role, that of redistributing resources. It taxes all employed agents with an income tax τ to pay for unemployment benefits b and fiscal transfers T. Additionally, the government fiscalizes firm profits which is equivalent to redistributing profits to workers proportionally to their earnings. It balances its budget period by period ensuring the following holds

$$\tau \int_{x^e} w(x^E)\, d\Psi^e + \Pi = b \int_{x^u} d\Psi^U + T \tag{14}$$

3.4 Aggregation

Matching Technology. There is one matching technology $M(\cdot)$ for all labor markets. If v_n and $searchers_n$ are the mass of vacancies and the mass of agents searching in labor market n, respectively, the matching function is

$$M(v_n, searchers_n) = \chi v_n^{1-\eta} searchers_n^{\eta}$$

The mass of agents searching on rung n is made up of agents from all labor markets

$$searchers_n = g(n|0) \int d\Psi^u + s \cdot \sum_{n'=1}^{N} g(n|n') \cdot \int_{x^e(n')} d\Psi^e_{n'}$$

where $x^e(n')$ indexes workers in labor market n'. Tightness in labor market n is $\theta_n = \frac{v_n}{searchers_n}$. Because of CRS, the vacancy-filling and job-finding rates are

$$q(\theta_n) = \chi \left(\frac{1}{\theta_n}\right)^{\eta} \qquad \lambda(\theta_n) = \theta_n \cdot q(\theta_n) \tag{15}$$

Equilibrium. The competitive equilibrium is a set of values for agents and firms $\{U, E, E^u, V, J, J^e\}$, policy functions $\{c^U, c^E, a^U, a^E, \Phi\}$, prices $\{r, r^K, w^U(\cdot), w^E(\cdot)\}$, and labor market tightnesses $\{\theta_n\}$ such that

1. Agents, firms, and the capitalist maximize their respective objectives.
2. The government balances its budget (14).
3. The asset market clears, $\underbrace{\int_{x^u} a(x^u)\, d\Psi^u + \int_{x^e} a(x^e)\, d\Psi^e}_{\text{HH wealth}} = \underbrace{\mathcal{P}(K)}_{\text{firm equity}}.$
4. The labor market clears $\underbrace{\sum_{n=1}^{N} \int_{x^e} z(x^e) \cdot p_n\, d\Psi^e}_{\text{labor supply}} = \underbrace{\sum_{n=1}^{N} \int L(k)}_{\text{labor demand}}.$
5. Free entry holds on each rung, that is $V(n) = 0$, or using 8, $q(\theta_n) = (1+r)\frac{\kappa p_n}{J_0(n)}$.

4 Wages: Generalized AOB for On-the-Job Search

Search-and-matching models with on-the-job search typically assume risk neutrality and no asset accumulation (see Cahuc, Postel-Vinay and Robin 2006). These assumptions are incompatible with the questions this paper tackles. I propose a new environment for on-the-job search that accommodates risk-aversion and asset accumulation and leads to a rich yet tractable solution for wages.

4.1 Summary

I propose a *generalized* alternating offer bargaining protocol for on-the-job search. This new environment builds on the variant of AOB developed in Christiano, Eichenbaum and Trabandt (2016) but, unlike their work, it accommodates on-the-job search in which two firms compete for one worker. The bargaining is characterized as a combination of Bertrand competition between firms and simple AOB between one worker and one firm. For unemployed workers, who negotiate with one firm, wages are determined by simple one-on-one AOB. For workers switching jobs, the way wages are determined depends on the productivities of the competing firms. If the firms have relatively similar productivities, wages are determined by Bertrand competition: the worker goes to the "better" firm and is paid a wage that delivers the maximum value the "worse" firm can provide. If the firms' productivities are far off, the "worse" firm becomes irrelevant and wages are determined by simple one-on-one AOB between the worker and the "better" firm.

4.2 Unemployed

Here I describe the negotiation between unemployed worker and firm of type n.

Players and Contract. Worker and firm negotiate a wage that persists until either the match dissolves or another firm tries to poach the worker. There are M (odd) sub-periods in which the players alternate proposing and considering offers. The firm makes offers in odd sub-periods, the worker in even sub-periods. If no agreement is reached by M, the worker goes back to unemployment and the firm remains vacant. If an agreement is reached, the worker starts producing at the agreed-upon wage.

Timing Assumptions. To minimize the theoretical and computational complications arising from curved utility and wealth accumulation, I make two assumptions.

Assumption 1. Shocks are realized at $m = 1$, interest accrues and the consumption/savings decision is made at $m = M$.

Assumption 2. If the worker and firm sign the contract at m, output and wages for the first period of the match are scaled down by the remaining number of sub-periods $\frac{M-m+1}{m}$.

The first is a timing assumption that allows to solve one rather than M consumption/savings problems. The second states that a worker at a new firm is assigned a task at the moment the contract is signed, m. Production will only take place in the remaining sub-periods, and so wages and output for that period are scaled down by $\frac{M-m+1}{M}$.

Payoffs. If worker and firm agree at m on wage w_m^n, their respective payoffs are

$$W_m^u\left(a, z, w_m^n\right) \equiv \max_{c, a'} u\left(c\right) + \beta \mathbb{E}\left[\sigma\left(0\right) U\left(a', z'\right) + \left(1 - \sigma\left(0\right)\right) E\left(a', z', w_m^n, n, 0\right)\right] \tag{16}$$

$$\text{s.t. } c + a' = Ra + \left(1 - \tau\right)\left[\frac{m-1}{M} b + \frac{M-m+1}{M} w_m^n\right] + T$$

$$J_m^u\left(a, z, w_m^n\right) \equiv \frac{M-m+1}{M}\left(zp_n\left[Zf\left(k\right) - r^K k\right] - w_m^n\right) + \frac{1}{1+r}\mathbb{E}\left[J\left(\psi_a, z', w_m^n, n, 0\right)\right] \tag{17}$$

Because the wage and profits are scaled down by the number of sub-periods remaining, both parties have an incentive to sign the contract as soon as possible.

Procedure and Equilibrium Actions. If it is their turn, players propose a wage, otherwise they evaluate the offer received. When a wage is rejected, if $m < M$ the bargaining continues into $m + 1$, if $m = M$ the bargaining breaks and the worker and firm remain unemployed and vacant, respectively. When a wage is accepted production begins.

At m odd the worker faces an outside option W_{m+1}^{wait}, the value to the worker if they reject the offer made by the firm and wait until $m + 1$. At $m = M$, this outside option is unemployment, $U\left(a, z\right)$, because the bargaining breaks down if the worker rejects the wage offer. The firm proposes w_m^n, the lowest wage the worker will not refuse, satisfying

$$W_m^u\left(a, z, w_m^n\right) = W_{m+1}^{\text{wait}} \tag{18}$$

and the firm draws value $J_m^u\left(a, z, w_m^n\right)$ which defines its outside option J_m^{wait} at $m - 1$.

At m even, the worker proposes wage w_m^n making the firm indifferent between accepting the wage and its outside option. This wage solves

$$J_m^u\left(a, z, w_m^n\right) = J_{m+1}^{\text{wait}} \tag{19}$$

The firm accepts and the worker draws value $W_m^u\left(a, z, w_m^n\right)$ from the match – this value defines the worker's outside option W_m^{wait} at $m-1$. The game is solved backwards and is resolved with the worker accepting the firm's offer at $m=1$.

4.3 Employed

I now describe the generalized AOB protocol. This provides a parsimonious solution to the negotiation that takes place when a worker of type (a, z, w, j) employed at firm of type n gets an offer from firm n'.

Players and Contract. The players are the worker, the incumbent firm n, and the poaching firm n'. They bargain to decide the allocation and the wage the worker will receive. As with the unemployed worker, firms make offers in odd sub-periods and workers in even sub-periods.

Additional Assumption. Assumptions 1 and 2 still hold. Assumption 2 takes on new meaning with job-switchers. Whenever job-switchers move to the poaching firm, production and wages are pro-rated and scaled down by $\frac{M-m+1}{M}$, where m indicates the sub-period the contract is signed. However, when a worker stays with the incumbent, output and wages are paid for the entire period regardless of m. This is because the worker is already under contract with the incumbent and so they are assigned a task in the first sub-period to complete over the entire period. As long as the worker stays with the incumbent, the full value of production is realized. Assumption 3 states what happens to the task started at the incumbent when the worker switches to the poacher.[9]

Assumption 3. If the worker signs a contract with the poacher at any time, the task at the incumbent remains undone and no output or wages incumbent are realized from it.

Procedure. All offers, those made by the firms to the worker and those made by the worker to the firms, are proposed simultaneously. Each party either rejects or accepts the offers made. The worker can accept at most one offer. If both firms accept the offer of the worker, I assume the incumbent wins. Whenever a wage offer is accepted, production starts between the worker and the "winning" firm. If all offers are rejected, the game moves on. At $m<M$ the bargaining continues into the next sub-period $m+1$; at $m=M$ the bargaining stops and all parties go back to their previous states: the worker and firm n remain engaged in production at the original wage w, and firm n' remains vacant.

[9] While not crucial, this assumption reduces the number of cases considered.

Payoffs. If at sub-period m an agreement is reached and the worker signs a contract with firm n at wage w_m^n, the payoffs for each party are as follows.

1. Firm n' remains vacant and has payoff $V(n') = 0$.
2. Firm n renegotiates the wage with the worker who, having been at the firm the entire period, produces for the entire period. The payoff to firm n is

$$J_m^n\left(w_m^n\right) = \left(y_n - r^K k - w_m^n\right) + \frac{1}{1+r}\mathbb{E}\left[\sigma(j)V\left(n\right) + \left(1-\sigma(j)\right)J\left(\psi_a, z', w_m^n, n, j+1\right)\right] \quad (20)$$

   where ψ_a is short-hand for the household's asset policy function.

3. The worker makes their consumption/savings decision based on the new wage w_m^n and their payoff is

$$\begin{aligned} W_m^n\left(w_m^n\right) &= \max_{c,a'} u\left(c\right) + \beta\mathbb{E}\left[\sigma\left(j\right)U\left(a', z'\right) + \left(1-\sigma\left(j\right)\right)E\left(a', z', w_m^n, n, j+1\right)\right] \\ \text{s.t.} \qquad & c + a' = Ra + (1-\tau)w_m^n + T \end{aligned} \quad (21)$$

Notice, I do not allow workers to be poached right after they sign a new contract.

In the case the worker signs a contract with firm n' at wage $w_m^{n'}$ in sub-period m, the payoffs are as follows.

4. Firm n' poaches the worker and starts producing from sub-period m onward. Thus, production and the wage rate paid are pro-rated. The payoff to firm n' is

$$\begin{aligned} J_m^{n'}\left(w_m^{n'}\right) &= \frac{M-m+1}{M}\left(y_{n'} - r^K k - w_m^{n'}\right) \\ &\quad + \frac{1}{1+r}\mathbb{E}\left[\sigma(0)V\left(n'\right) + \left(1-\sigma(0)\right)J\left(\psi_a, z', w_m^{n'}, n', 1\right)\right] \end{aligned} \quad (22)$$

5. Firm n becomes vacant and has payoff $V(n) = 0$.
6. The worker makes their consumption/savings decision based on the new wage $w_m^{n'}$ but, in the first period, this is pro-rated. Their payoff is

$$\begin{aligned} W_m^{n'}\left(w_m^{n'}\right) &= \max_{c,a'} u\left(c\right) + \beta\mathbb{E}\left[\sigma\left(0\right)U\left(a', z'\right) + \left(1-\sigma\left(0\right)\right)E\left(a', z', w_m^{n'}, n', 1\right)\right] \\ \text{s.t.} \qquad & c + a' = Ra + \frac{M-m+1}{M}(1-\tau)w_m^{n'} + T \end{aligned} \quad (23)$$

The fact that, at m, the incumbent produces for the entire period while the poacher only produce for the remaining sub-periods is important. This asymmetry implies that the poacher is "impatient" relative to the incumbent and is willing to compensate the worker with a higher wage to sign the contract immediately rather than wait and lose output and profits by moving on to the next bargaining sub-period. On the contrary, the incumbent makes the same output regardless of when it signs on the worker.

Definitions and Results. Before considering the actions pursued I define two key concepts. First, the highest wage a firm is willing to pay the worker, that is the wage making firms indifferent between hiring and posting a vacancy; second, the corresponding value the worker gets from this wage.

Definition 4.1. Denote by $\overline{w}_m^n$ and $\overline{w}_m^{n'}$ the *break-even wages* firms n and n' are able to pay the worker at sub-period m. These wages satisfy:

$$J_m^n\left(\overline{w}_m^n\right) = V\left(n\right) = 0 \qquad \text{and} \qquad J_m^{n'}\left(\overline{w}_m^{n'}\right) = V\left(n'\right) = 0$$

Result 1. The break-even wage the incumbent n can offer is independent of m, the one the poacher n' can offer is strictly decreasing in m.

$$\overline{w}^n := \overline{w}_1^n = \cdots = \overline{w}_M^n \qquad \text{and} \qquad \overline{w}_1^{n'} > \cdots > \overline{w}_M^{n'}$$

Proof. See appendix A.2. □

Definition 4.2. Denote by $\overline{W}_m^n$ and $\overline{W}_m^{n'}$ the *break-even valuations* the worker can extract from firms n and n'. They satisfy:

$$\overline{W}_m^n = W_m^n\left(\overline{w}_m^n\right) \qquad \text{and} \qquad \overline{W}_m^{n'} = W_m^{n'}\left(\overline{w}_m^{n'}\right)$$

Result 2. The break-even valuation the worker can extract from the incumbent n is independent of m, the one they can extract from the poacher n' is strictly decreasing in m.

$$\overline{W}^n := \overline{W}_1^n = \cdots = \overline{W}_M^n \qquad \text{and} \qquad \overline{W}_1^{n'} > \cdots > \overline{W}_M^{n'}$$

Proof. See appendix A.2. □

These results make clear that the incumbent firm can offer the worker the same value regardless of the sub-period m the contract is signed; in contrast, the poacher becomes more and more constrained in the value it can provide to the worker as m increases.

Equilibrium Actions. In odd sub-periods m, it is the firms' turn to bid for the worker. Firms bid simultaneously offering the minimal wage that can attract the worker conditional on not paying more than their break-even valuations.[10] Suppose the value the worker gets by waiting until the next sub-period, $m+1$, is W_{m+1}^{wait}. If a firm wants to attract the worker, it must offer the maximum between the valuation the other firm has for the worker and W_{m+1}^{wait}. A penny less and either the worker accepts the other firm's offer or the worker decides to move to sub-period $m+1$. However, firms must also not offer the worker more than their own break-even valuations as they would otherwise prefer posting a vacancy to hiring the worker. This results in these simple rules for the offers made by firms n and n', respectively:

$$
\begin{aligned}
W_m^{n,\text{bid}} &= \min\left\{\max\left\{W_{m+1}^{\text{wait}}, \overline{W}_m^{n'}\right\}, \overline{W}^n\right\} &(24)\\
W_m^{n',\text{bid}} &= \min\left\{\max\left\{W_{m+1}^{\text{wait}}, \overline{W}^{n}\right\}, \overline{W}_m^{n'}\right\} &(25)
\end{aligned}
$$

The inner maximization is required for the firm to attract the worker. The outer minimization is required for the firm to find it profitable to attract the worker.

In even sub-periods m, it is the worker who makes offers to the firms. The worker first evaluates the wages that make each firm indifferent between hiring the worker and moving on to the next sub-period.[11] The wage $w_m^{n'}$ makes the poacher indifferent between accepting today and moving on to $m+1$ and receiving value J_{m+1}^{wait}. This wage solves[12]

$$
\frac{M-m+1}{M}\left(y_{n'} - r^K k - w_m^{n'}\right) + \frac{1}{1+r}\mathbb{E}\left[\sigma(0)V\left(n'\right) + (1-\sigma(0))\, J\left(\psi_a, z', w_m^{n'}, n', 0\right)\right] = J_{m+1}^{\text{wait}}
$$

The value to the worker corresponding to this wage is $W_m^{n'}\left(w_m^{n'}\right)$. This is exactly what occurs in simple one-on-one AOB. However, there is an additional event to consider. The incumbent firm n could beat this offer. In this case the worker may be able to extract an even higher wage from the poacher n', otherwise the worker would stay at the incumbent and the poacher would remain vacant. This means the worker can extract from the poacher n' the maximum between the worker's outside option, $\overline{W}_m^n$ (i.e. the break-even valuation the incumbent is able to afford), and the value granted by one-on-one negotiation, $W_m^{n'}\left(w_m^{n'}\right)$, as long as this does not exceed the break-even valuation of the poacher,

[10] This interpretation corresponds to the firms bidding for the worker in a sealed-bid first price auction.

[11] This interpretation is equivalent to the worker first making firm n' indifferent between accepting and moving on to the next period and then asking firm n to match that offer.

[12] If n' is not able to poach the worker at $m+1$, J_{m+1}^{wait} is the value of a vacancy, that is 0, and $w_m^{n'} = \overline{w}_m^{n'}$.

$\overline{W}_m^{n'}$. Mathematically this value is expressed as

$$W_m^{n',\text{bid}} = \min\left\{\max\left\{W_m^{n'}\left(w_m^{n'}\right), \overline{W}^n\right\}, \overline{W}_m^{n'}\right\} \tag{26}$$

where the inner maximization ensures the poacher attracts the worker and the outer minimization ensures it does not pay the worker more than its break-even valuation.

The scenario is simpler when dealing with the incumbent n. While the poacher n' can offer the worker more at m than at $m+1$, and in fact compensates the worker for not waiting an extra sub-period, the incumbent n would actually prefer waiting until $m+1$ because it would see no loss in output but would have to compete with a weaker offer from the poacher n' (as per result 2). This worker proposes firm n a wage delivering value

$$W_m^{n,\text{bid}} = \min\left\{\max\left\{W_{m+1}^{\text{wait}}, \overline{W}_m^{n'}\right\}, \overline{W}^n\right\} \tag{27}$$

The earlier interpretation applies here: the incumbent pays the worker the highest between the poacher's break-even value and the worker's waiting value, as long as this does not exceed the break-even value the incumbent can afford. For a thorough explanation of the strategies see appendix B.

Final Outcomes. The bargaining is resolved in the first sub-period. The allocation rule is simple: the worker chooses the firm *able* to provide them with the highest value.

Result 3 (Allocations)**.** The worker is poached by firm n' if and only if $\overline{W}_1^{n'} > \overline{W}^n$. Otherwise the worker is retained by firm n.

Without tenure workers go to the most productive firm because it is always able to offer the highest wage and hence value. When workers value tenure, they will go to the poacher only if it is sufficiently more productive than the incumbent. This is because the poacher must pay the worker a premium to compensate for the lost job stability tenure endows them with. As figure 2 shows, when tenure is included, workers make fewer job-switches because the set of attractive poachers shrinks.

Result 4 (Wages)**.** The values and wages workers will agree to are one of the following:

(1) if $\overline{W}_1^{n'} < W^n(w)$, the worker is retained by the incumbent n at the original wage w.

(2) if $W^n(w) < \overline{W}_1^{n'} \leq \overline{W}^n$ the worker is retained by the incumbent n at wage $w^{n,B} > w$ that delivers the worker the break-even valuation of n', satisfying $W_1^n\left(w^{n,B}\right) = \overline{W}_1^{n'}$.

(3) if $\overline{W}_2^{n'} \leq \overline{W}^n < \overline{W}_1^{n'}$ the worker is poached by n' at wage $w^{n',B}$ that delivers the worker the break-even valuation of n, satisfying $W_1^{n'}\left(w^{n',B}\right) = \overline{W}_1^n$.

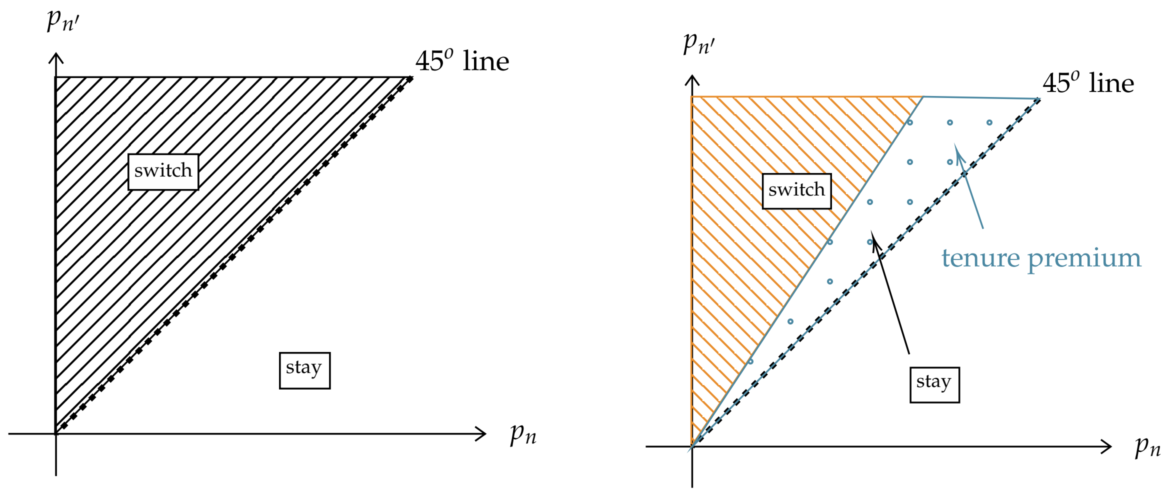
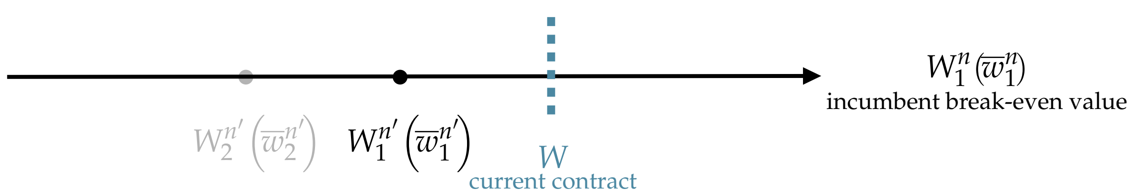
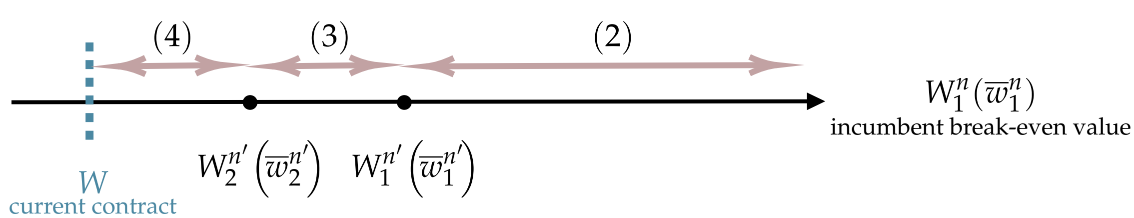


Figure 2: Allocation rules for fixed worker type by productivity in an economy with (second panel) and without (first panel) tenure.

(4) if $\overline{W}_2^{n'} > \overline{W}^n$ the worker is poached by firm n' and the wage $w^{n,AOB}$ is agreed upon by one-on-one negotiation between the poacher n' and the worker. This negotiation starts at $m = 1$ and lasts until sub-period m^{end}, the last sub-period in which the break-even valuation of n' dominates that of n. That is, m^{end} satisfies $\overline{W}_{m^{\text{end}}}^{n'} > \overline{W}^n \geq \overline{W}_{m^{\text{end}}+1}^{n'}$ with $\overline{W}^n$ being the worker's outside option and 0 (i.e. posting a vacancy) the outside option of firm n' at m^{end}. If no such m^{end} exists for $m^{\text{end}} \in \{1, \ldots M\}$ then $m^{\text{end}} = M$.



(a) Scenario 1.



(b) Scenario 2-4.

Figure 3: Bargaining outcomes and cutoffs. (a) Scenario 1: the current contract beats break-even value the poacher can offer. (b) Scenario 2-4: the break-even value the poacher can offer beats the current contract.

These outcomes have a simple interpretation highlighted in Figure 3. In case (1) shown in figure 3a, the poaching firm is so much less productive than the incumbent that any offer it makes is irrelevant and so the worker stays at the incumbent with the same wage. In cases (2) and (3), shown in figure 3b, the two firms have relatively similar productivities and so they threaten each other's prospects of getting the worker and Bertrand compete for the worker. In case (4), shown in figure 3b, the poacher is so productive that any offer the incumbent makes is irrelevant and so the worker and poacher bargain one-on-one.

Case (4) is subtle. The break-even value of the poacher is so much higher than that of the incumbent that even if negotiations were to move into sub-period $m = 2$ the poacher would win over the worker. This makes the poacher "compete" against time rather than against the incumbent. While in (3) waiting until $m = 2$ means the poacher loses the worker, in (4) waiting means losing profits but still poaching the worker. The worker knows this and negotiates one-on-one with the poacher. This one-on-one negotiation will go on until the incumbent's offer becomes relevant again which, per result 2, will happen at some m^{end} such that $\overline{W}^{n'}_{m^{\text{end}}} > \overline{W}^{n} > \overline{W}^{n'}_{m^{\text{end}+1}}$.[13] The bilateral negotiation is solved backwards starting from m^{end}. If it is the worker's turn to make the offer, they propose $\overline{w}^{n'}_{m^{\text{end}}}$ extracting the break-even wage from n'; if it is the firm's turn to make the offer it proposes a wage delivering the worker the break-even value of the incumbent, $\overline{W}^{n}$. As before, the process carries on backwards until $m = 1$ as in the standard alternating offer bargaining with two players.

Note that substituting Bertrand for case (4) would lead to potentially unrealistic predictions because workers moving from very low to very high productivity firms would experience only relatively small wage increases as the poacher would only need to match the break-even value the low-productivity incumbent can provide. This generalized alternating bargaining protocol for on-the-job search provides a theoretical foundation for *not always* having wages determined à la Bertrand.

5 Calibration

The model is calibrated to match key moments of the US economy with particular attention to the labor market and the wealth distribution. I start by estimating the set of parameters that pertains to the risk incurred when workers switch jobs.

[13]Note this is bound to happen at some m because at M the $\overline{W}^{n'}_{M+1} = 0$.

5.1 Job-Switching Risk Estimation

At the heart of the model lies the risk that workers incur when moving from one job to another. To estimate this risk I use the SIPP and show that when a worker moves to a new job they face a probability of job loss that, over the first five quarters following the move, is 7.4 percentage points higher than if they had not switched jobs.

I quantify this risk using an event study similar to that of Davis and Von Wachter (2011). The goal is to capture the *additional* probability that a worker will suffer an unemployment spell after switching jobs. To do this, I follow workers in the SIPP panel, tracking their job switches (by using the identifier of the firm at which they are employed) and their moves from employment into unemployment. The linear probability model I run is

$$\mathbb{1}\left(\mathrm{EU}_{i,t}\right) = \sum_{k=-1}^{14} \boldsymbol{\theta_k D^k_{i,t}} + \underbrace{\alpha_i}_{i\text{-FE}} + \underbrace{\beta_t}_{t\text{-FE}} + \Gamma X_{i,t} + \varepsilon_{i,t} \tag{28}$$

where $\mathbb{1}\left(\mathrm{EU}_{i,t}\right)$ are realizations of worker i's moves from employment in period t to unemployment in period $t+1$, $\boldsymbol{D^k_{i,t}}$ are a series of dummy variables that take on value 1 if worker i at time t switched jobs k months back, α_i and β_t are individual and time fixed effects, respectively, and $X_{i,t}$ are time-varying controls for individual i, namely age and industry of occupation. θ_k captures the additional probability of falling into unemployment that a worker who switched jobs k periods back faces compared to a similar worker who did not switch. Because, as I discuss in the next sub-section, in the calibrated model workers only move to higher-paying jobs, I make this same restriction in the estimation. Figure 4 illustrates the θ's, aggregated quarterly, in the five quarters after a job switch.

Two aspects emerge: the estimates are positive, meaning workers who switch jobs face a subsequently higher risk of unemployment, and they are persistently so, decaying towards 0 only after fifteen months from the start of the new job. The orange dotted line shows the model equivalent when choosing parameters π^H and α^L to match the data.

Beyond being statistically significant, these results are economically significant. Over the first five quarters following a job-to-job transition, the probability of a worker falling into unemployment increases by 7.4 percentage points.[14] Considering that, over the same five quarter interval, the probability the average US worker falls in unemployment is roughly 18%, these estimates indicate a considerable increase in risk: workers who switch jobs face a one third higher likelihood of being hit by an unemployment spell in the fifteen months after the job switch.

[14] The estimates are highest during recessions and for low-wealth workers.

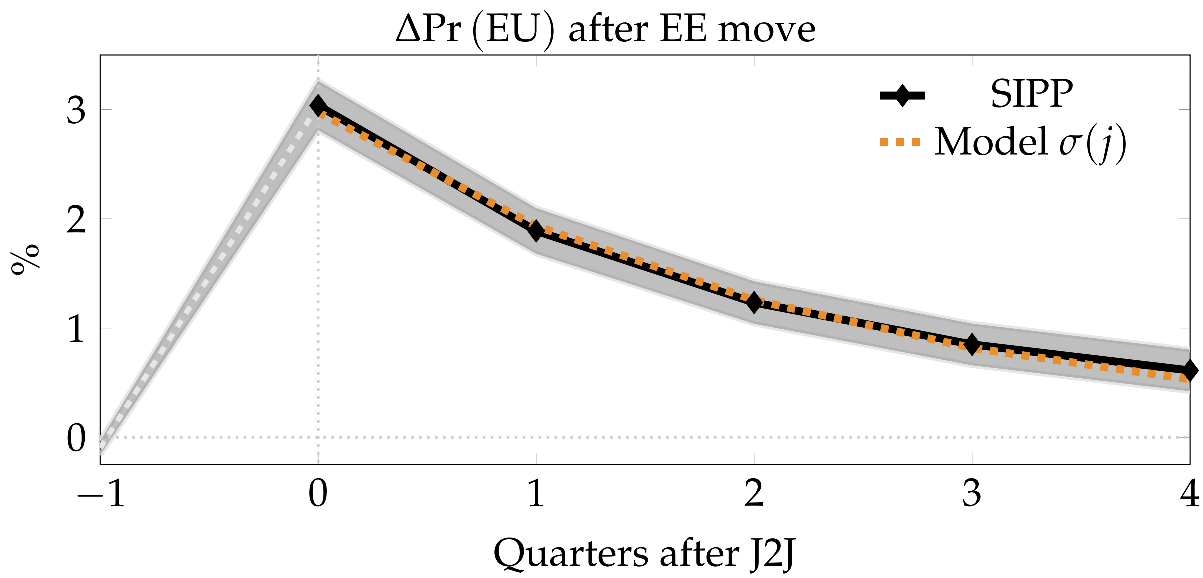


Figure 4: Change in probability of job loss after a J2J transitions. Estimated using SIPP.

In the next section, I study the economic consequences of this increased risk of job-loss through the lens of my model but before doing so I discuss the calibration of the remaining model parameters.

5.2 Aggregate Moments

The other model parameters are calibrated via SMM to match key moments of the US economy. Table 2 displays the parameters used in the model.

The model displays CRRA utility with risk-aversion parameter $\gamma = 2$. To match the empirical wealth distribution I employ two approaches. First, I use permanent discount factor heterogeneity as in Krusell and Smith (1998), with $\beta^L = 0.9565$ and $\beta^H = 0.9835$. Second, I calibrate the firm productivity grid with $p_n \in \{0.67, 0.74, 0.90, 1, 1.11, 1.28, 3.00, 9.03\}$ including two "superstar" productivity rungs to mimic the "superstar" income states in Kindermann and Krueger (2022).

The production parameters are standard in the literature. I use a capital share $\alpha = 0.3$ and a quarterly depreciation rate of capital $\delta = 2.5\%$. The elasticity of investment to q is set to 4 as in Auclert et al. (2021).

I assume that in steady state the government pays no lump sum transfers to agents ($T = 0$) but pays unemployment benefits $b = 0.07$ where this is set to match the ratio of unemployment expenditures to gross domestic income of 0.4%.

The labor market parameters help match quarterly moments of the labor market,

	Parameter (Quarterly Frequency)	Value
Household		
$u(c)$	Utility func.	$\frac{c^{1-\gamma}}{1-\gamma}$, $\gamma = 2$
(β^L, β^H)	Discount factor	$(0.9565, 0.9835)$
Firm		
α	Capital share	0.3
δ	Capital depreciation	2.5%
ϵ_I	Elasticity of I to q	4
Fiscal		
b	Unemp. benefits	0.07
T	Lump sum transfer	0
Labor Market		
s	On-the-job search intensity	0.45
$g(k+1\|k)$	Prob. search on next rung	1
κ	Vacancy cost	1.1
η	Matching elasticity	0.2
χ	Matching efficiency	0.67
M	Bargaining periods	3
α^{EV}	Std. of taste shocks	$1/100$
π^G	Prob. high potential	0.915
α^L	Prob. informative (L) signal	0.35

Table 2: Model Parameters.

	Wealth Share Owned by Quintile (%)					Labor Market Moments			
	Q1	Q2	Q3	Q4	Q5	EE rate	unemp. rate	job finding rate of unemp.	UI to income ratio
Model	1.15	5.15	9.54	17.3	66.8	4.15%	5.70%	48.91%	0.327%
Data	-1.04	0.68	6.85	18.21	75.3	4.14%	6.02%	55.70%	0.396%

Table 3: Wealth and labor market moments in model steady state and data. First half of the table has net worth share owned by each quintile of the wealth distribution. Second half of the table has main labor market indicators. The unemployment insurance to income ratio is computed as total federal unemployment benefits over gross national income taken from the BEA. Sources: PSID, SIPP, BEA.

specifically the job-finding probability, 56%, the unemployment rate, 6.0%, and the job-switching rate, 4.1%.[15] The model parameters that are most useful for matching these targets are the intensity of search when employed, $s = 0.45$, the vacancy posting cost per unit of firm productivity, $\zeta = 1.1$, the matching elasticity, $\eta = 0.2$, and the matching efficiency, $\chi = 0.67$. I impose that agents and firms bargain over $M = 3$ sub-periods, cor-

[15]In the model this is equivalent to the fraction of job-switchers out of the employed workers excluding the last rung since these workers have nowhere to go. Relaxing that distinction makes little difference, bringing the EE rate from 4.16% to 3.95%, since very few workers are situated there.

responding to monthly offers and counteroffers. π^G and α^L, the probability a worker has high potential and the probability the firm gets an informative signal, are chosen to match the empirical additional probability of job-loss estimated in the previous sub-section.

Table 3 shows the targeted moments and the model equivalent both for the wealth distribution and for the major labor market indicators. Data moments are computed over the 1996-2013 period.

5.3 Job-Switching Sensitivity to Wealth

I now test the validity of the calibrated model by checking whether it can replicate a relevant untargeted empirical moment, the sensitivity of job-switching to wealth. The precautionary job-keeping motive emerges as a positive relationship between wealth and the probability of switching jobs and should thus be reflected in a positive elasticity of job-switching to wealth. To compute this sensitivity of job-switching to wealth in the SIPP, I run the following regression

$$\mathbb{1}\left(\mathrm{EE}_{i,t}\right) \quad = \quad \beta_0 + \boldsymbol{\beta_1}\frac{\mathrm{Wealth}_{i,t}}{\mathrm{Income}_{i,t}} + \vec{\gamma}X_{i,t} + \alpha_i + \delta_t + \varepsilon_{i,t} \tag{29}$$

On the left-hand side are realizations of job-switches for worker i at time t (1 if the worker switches, 0 otherwise). On the right-hand side are worker i's wealth-to-income ratio at time t, time and individual fixed effects, as well as a polynomial in age and industry of occupation. I run the same regression using the model steady state by simulating individual employment and wealth paths for agents in the model.[16] Repeating this regression on both the SIPP and the model-simulated data for each decile of the wealth-to-income distribution leads to figure 5 where in black are the data, and in solid orange the model. Additionally, in the dotted orange, is the model in which I control for the rung workers are at, which inherently accounts for the types of incumbent and poacher switchers face.

The model does a good job matching the untargeted empirical sensitivities. In both the model and the data, high-wealth workers' job-switching decisions essentially do not depend on their wealth-to-income ratio. These workers are not very sensitive to risk as they already have the means to self-insure and more wealth does not alter their risk-reward calculus when confronted with switching jobs. On the contrary, for workers with low wealth-to-income ratios the sensitivity is positive. In the data, for workers in the first decile, an extra annual income worth of wealth increases the probability of switching jobs by 13 percentage points, a large increase given the average quarter on quarter probability

[16] I only control for individual fixed effects when running the regression in the model.

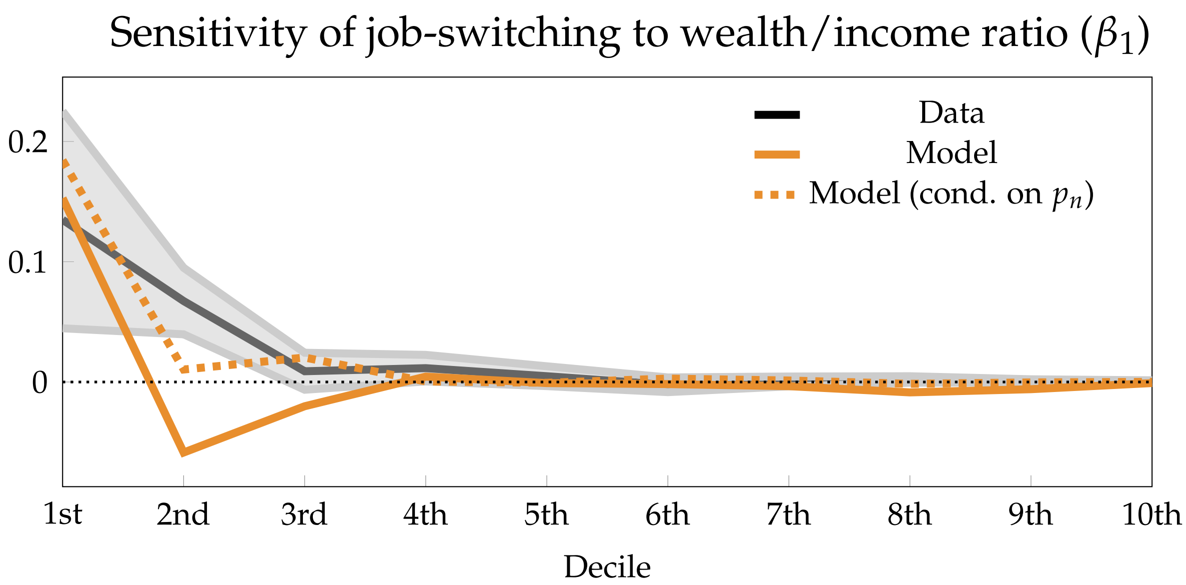


Figure 5: Job-switching elasticity to wealth-to-income ratio in the data (black) and the calibrated model (orange). The regression is run for each decile of the wealth-to-income distribution.

of switching jobs in the US is roughly 4 percent. The figure shows that this sensitivity is montonically decreasing. This is consistent with precautionary job-keeping: the lower wealth a worker has, the higher the marginal utility, the more effectively risk-averse, and the less willing they are to switch jobs.

6 Mechanisms and Matching the Business Cycle Moments

In this section I detail how a job-loss probability declining in tenure gives rise to (i) the precautionary job-keeping motive and (ii) the tenure-wealth correlation. I also explain how these mechanisms allow the model to match the empirical cyclical moments of the job-switching and job-losing rates across the wealth distribution.

6.1 Precautionary Job-Keeping

Precautionary job-keeping is a causal mechanism linking workers' wealth to their job-switching decision. At its heart is the trade-off between higher wages and lost job-stability that workers experience when they switch jobs. Workers with different wealth respond differently to this trade-off, with low-wealth workers valuing job-stability relatively more.

Equation 4 displays the trade-off workers face when switching jobs: they earn higher wages when moving to a higher productivity firm but this comes at the cost of lost tenure and consequently higher unemployment risk. As the expressions implies, there is an asset threshold $a^* \in [\underline{a}, \infty]$ such that worker of type (z, w, n, j) moves to the new firm only if their wealth exceeds this threshold ($a \geq a^*$). In other words, low-wealth workers are particularly sensitive to the unemployment risk switching entails. This is because, with little wealth, they have limited means to insure against eventual unemployment spells. High-wealth workers, in contrast, can use their wealth to smooth consumption if they are hit by an unemployment spell. Thus it is low-wealth workers who have a stronger precautionary incentive not to switch to a new job.

Aggregating the individual policy functions across workers in the economy delivers a probability of switching jobs conditional on receiving an offer that is increasing in assets as shown in the solid black line of figure 6.[17] Additionally, figure 6 displays the steady state distribution of workers across assets (green solid line). The vertical green dotted line separates the bottom from the top half of the wealth distribution. Workers in the bottom half face a much steeper probability of switching curve than workers in the top half. This is entirely a reflection of diminishing marginal utility: changes in wealth have little impact on marginal utility for high-wealth workers but large impacts for low-wealth workers. This in turn means that for low-wealth workers the probability of switching jobs is very sensitive to changes in wealth.

When a recession hits the economy it depletes workers' wealth. The distribution of workers following a simulated recession is depicted in figure 6 (red dashed line). An important effect of this shift is that, after a loss in wealth, low-wealth workers, who face the steep side of the probability of switching curve, experience a large fall in their conditional probability of switching. This implies their overall job-switching probability falls considerably. In contrast, high-wealth workers, who face the flat side of the curve, see little change in their conditional job-switching probability following a loss in wealth. These forces explain the higher cyclical volatility of the job-switching rate at the bottom of the wealth distribution relative to that at the top.

6.2 Tenure-Wealth Correlation

The tenure-wealth correlation is a mechanism due to the model's dynamic selection forces that links workers' wealth to their job-loss probability.

At its heart lie two facts: recessions reshuffle workers towards low tenure jobs, and this

[17] There is a caveat to this. Because workers are subject to taste shocks when choosing to switch jobs, as wealth increases these taste shocks become relatively more important and, as assets grow large the probability of switching is completely determined by the taste shocks. For all practical purposes, all but the very wealthiest agents in the model face an upward-sloping probability of switching jobs curve.

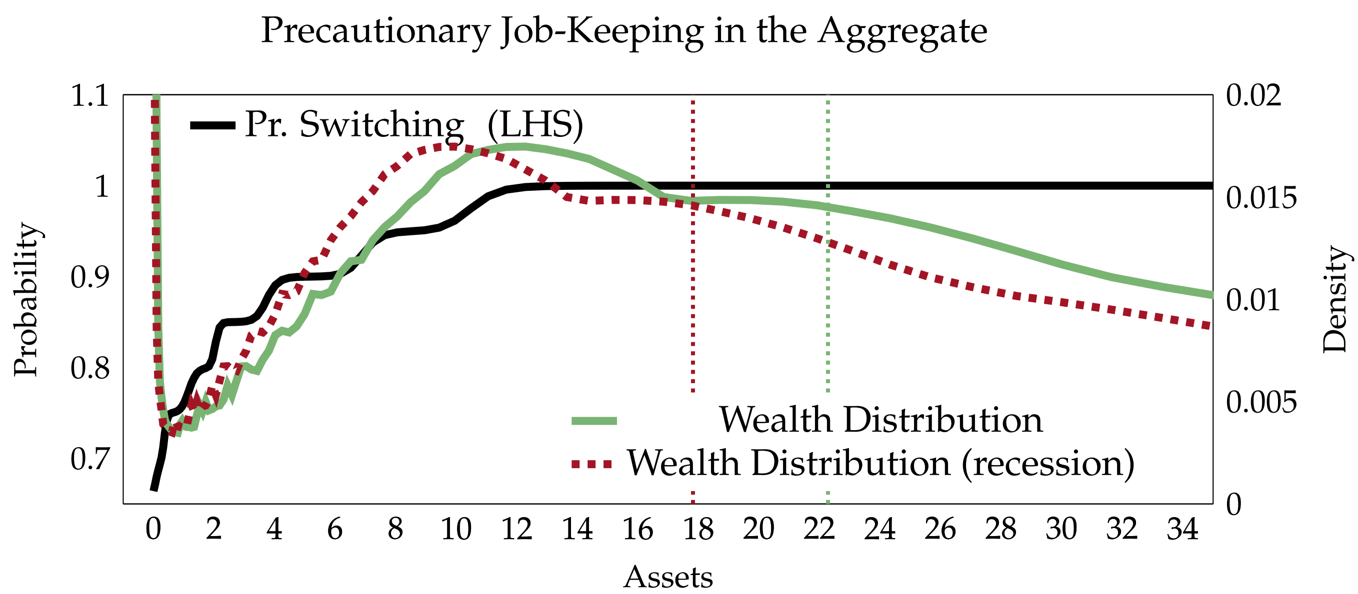


Figure 6: Model derived job-switching probability conditional on offer (black, RHS), steady state wealth distribution (green), and recession wealth distribution (red). Dotted lines indicate median wealth thresholds in steady state (green) and recession (red). Figure is truncated at assets of 35 and density of 0.02.

reshuffling mainly affects low-wealth workers. Because low-tenure means higher job-loss probability, low-wealth workers become more likely to lose their jobs after recessions.

After recessions hit an economy, the unemployment pool grows. As the economy recovers, unemployed workers slowly re-enter the labor market but these workers, because they are newly employed, occupy low-tenure jobs. This translates into a shift in the distribution of workers towards lower tenure jobs with higher probability of job loss.

Low-wealth workers are overwhelmingly subject to this redistribution because they make up a larger share of the unemployment pool, especially during recessions. There are two reasons for this. The first is straightforward. During a recession the duration of unemployment increases and so the unemployed run down their savings more than in normal times, *de facto* becoming low-wealth.

The second reason is more subtle. In any given period, low-wealth workers make up a larger share of job-losers because they tend to occupy relatively low-tenure jobs. The fact that low-wealth workers are more distributed in low-tenure jobs may seem puzzling. After all, low-wealth workers are more susceptible to the precautionary job-keeping motive and value tenure the most. However, this logic is trumped by the model's dynamic selection forces. Low-wealth workers tend to have low wealth precisely because they have had an unfortunate labor market history, having recently suffered either a long or multiple unemployment spells. Because they are more likely to have recently experienced unemployment, *employed* low-wealth workers are in turn more likely to be in relatively newer jobs with low tenure. This selection leads to the correlation between wealth and tenure and, in turn, between wealth and job-loss probability.

In sum, because low-wealth workers are reshuffled towards low-tenure jobs that are likelier to lead to layoffs, they see a larger increase in their probability of falling into unemployment following recessions.

6.3 Empirical Moments

The first model success is its ability to capture distributional differences in the cyclicality of the job-switching (EE), job-losing (EU), and unemployment (u) rates across wealth.

I start by estimating a joint stochastic process for productivity, Z, and the common vacancy filling cost, κ. These processes are

$$\kappa_t - \kappa^* = \rho^{\sigma}\left[\kappa_{t-1} - \kappa^*\right] + \epsilon_t^{\kappa} \tag{30}$$

$$\log\left(Z_t\right) - \log\left(Z^*\right) = \rho^{Z}\left[\log\left(Z_{t-1}\right) - \log\left(Z^*\right)\right] + \epsilon_t^{Z} \tag{31}$$

where

$$\begin{pmatrix} \epsilon_t^{\kappa} \\ \epsilon_t^{Z} \end{pmatrix} \sim \mathcal{N}\left(\vec{0}, \Sigma = \begin{pmatrix} Var(\kappa) & Cov(\kappa, Z) \\ Cov(\kappa, Z) & Var(Z) \end{pmatrix}\right) \tag{32}$$

Using the sequence-space Jacobian approach developed in Auclert et al. (2021), I compute transition dynamics for the model and estimate these processes to match the headline standard deviations and persistence of the job-switching, job-losing, and unemployment rates.[18] With the estimated processes at hand, I compute the same moments for the bottom and top of the wealth distribution separately. In table 4 I show the standard deviations in the data, in the model, and in a "naïve" model. The naïve model is the benchmark model except there is no decreasing job-loss probability in tenure. Instead, this is constant, $\sigma\left(j\right) = \sigma$, and calibrated to match the unemployment rate in the benchmark model. A constant job-loss probability suppresses any role tenure plays in the benchmark model, therefore neutralizing both precautionary job-keeping and the tenure-wealth correlation. Table 4 shows that the benchmark model almost exactly matches the headline standard deviation and its dispersion for the EE rate. While the model cannot capture the volatility in the EU rate, just like the data, it implies a twice as volatile EU rate for low-wealth

[18] I estimate these processes by simulating a series of short-lived shocks . This is because, to compute the distributional moments, I must compute non-linear dynamics of an economy that returns to steady state.

workers than for high-wealth workers. Finally, the model performs relatively well even when it comes to the unemployment rate. Without ad-hoc fixes to the Shimer puzzle, the model accounts for a great deal of the unemployment volatility in the data and implies that low-wealth workers experience a more volatile unemployment rate than high-wealth workers albeit not to the same extent as the data. On the contrary, the naïve model is unable to match these moments. The naïve model is doomed to fail when it comes to the job-losing rate (EU) since, by construction, the model has a constant job-loss probability for all workers. However, even in the case of the job-switching rate (EE), the naïve model displays little difference across wealth. In the next section I show why capturing these moments is important to understand aggregate economic dynamics in the labor market.

Standard Deviation (by wealth)									
	Data			Model			Naïve Model		
	all	low	high	all	low	high	all	low	high
EE	1.19	1.54	0.99	1.21	1.51	0.89	1.20	1.22	1.11
EU	1.20	1.55	0.91	0.08	0.10	0.05	0.00	0.00	0.00
u	1.57	2.45	1.03	0.83	0.88	0.79	0.88	1.00	0.86

Table 4: Standard deviations of job-switching, job-losing, and unemployment rate across the distribution of net worth. The series are Hamilton-filtered. All data are computed using SIPP 1996-2013.

7 Results

Precautionary job-keeping and the low-tenure trap contribute to the slower earnings recoveries experienced by low-wealth workers relative to high-wealth workers. Taken together, these two mechanisms explain 50 percent of the earnings gap observed after the Great Recession. In addition, through precautionary job-keeping, the model explains the Great Reallocation, the sudden increase in job-switching the US labor market experienced following the Pandemic, via the large government stimulus issued over this period.

7.1 Great Recession Earnings Recovery

The 2007-09 recession was the largest to hit the United States since the Great Depression. However, this recession did not affect workers equally: labor earnings for low-wealth workers fell much more than for high-wealth workers. Here, I assess how the model can speak to the heterogeneous earnings dynamics across the wealth distribution.

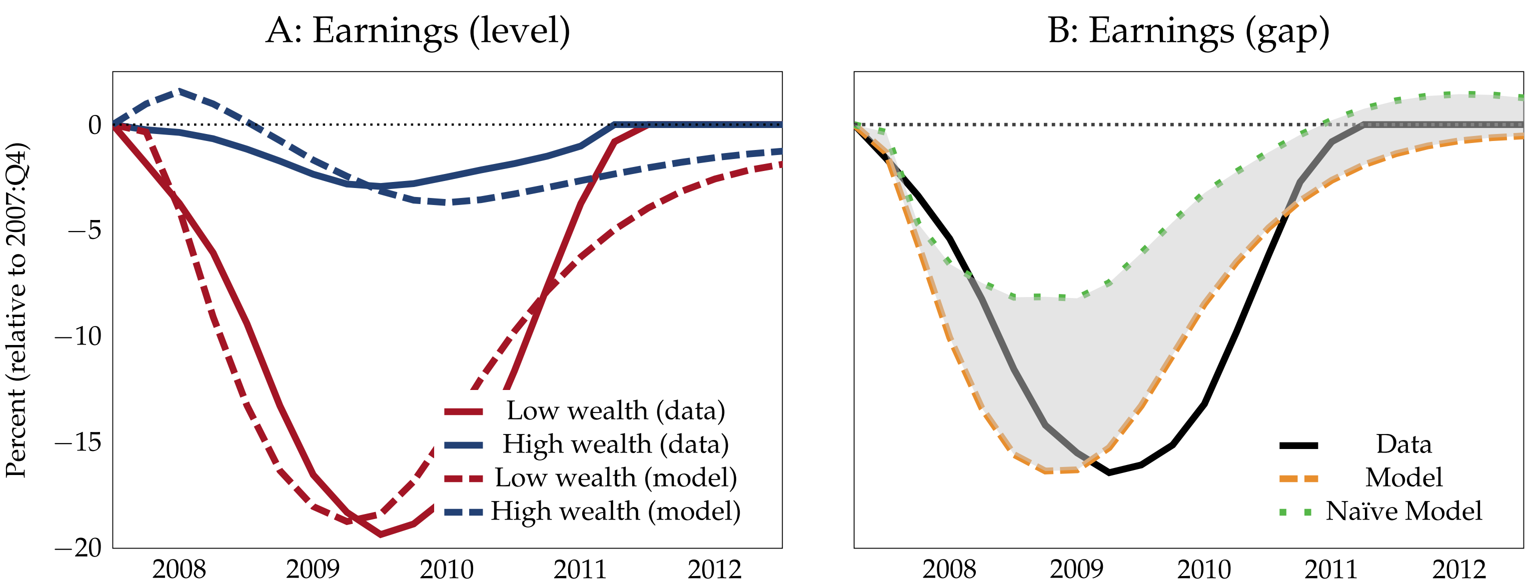


Figure 7: (a) Earnings growth for low- (red) and high-wealth (blue) workers in the data (solid) and the model (dashed). (b) Earnings gap in data (black), in benchmark model (orange), in naïve model (green).

I start by estimating paths of shocks for productivity, Z_t, and for the vacancy-filling cost, κ_t, to match the headline unemployment and earnings dynamics observed during the Great Recession. Subjecting the model to these shocks I compute labor earnings for low- and high-wealth workers. Panel A of figure 7 plots the empirical[19] (solid) and model-implied (dashed) earnings dynamics for high- (blue) and low-wealth workers (red). The lines show the percent change relative to 2007:Q4 earnings. The model captures the empirical dynamics in earnings by wealth.

Next, I ask how much of this earnings gap is due to the novel forces of the model, precautionary job-keeping and the tenure-wealth correlation. To answer this question I compare the benchmark model to the naïve model introduced earlier. This is shown in panel B of figure 7 which plots the earning *gaps* between low- and high-wealth workers. The solid black line shows the empirical gap, the dashed orange shows the gap implied by the benchmark model, and the dotted green shows the gap implied by the naïve model.

The naïve model can explain 44 percent of the empirical earnings gap. Even with no tenure, selection forces lead to a widening earnings gap by having "lucky" workers, those who find jobs and climb the job ladder, earn high wages and accumulate wealth. The area between the green dotted and orange dashed lines indicates the additional contribution of my model. This area corresponds to over half of the empirical earnings gap

[19]Earnings are de-trended using the Hamilton filter and kept constant once the new cycle begins.

and essentially allows to explain the entire gap experienced after the Great Recession.

To understand what lies behind these earnings dynamics, it is useful to consider the job-switching rates of high- and low-wealth workers in figure 8. After the start of the recession, the job-switching rate of low-wealth workers falls by twice as much as that of high-wealth workers. This is precisely in line with the theory of precautionary job-keeping espoused in this paper. The earnings of low-wealth workers are falling behind those of their high-wealth peers in part because they are not climbing up the job ladder at the same rate.

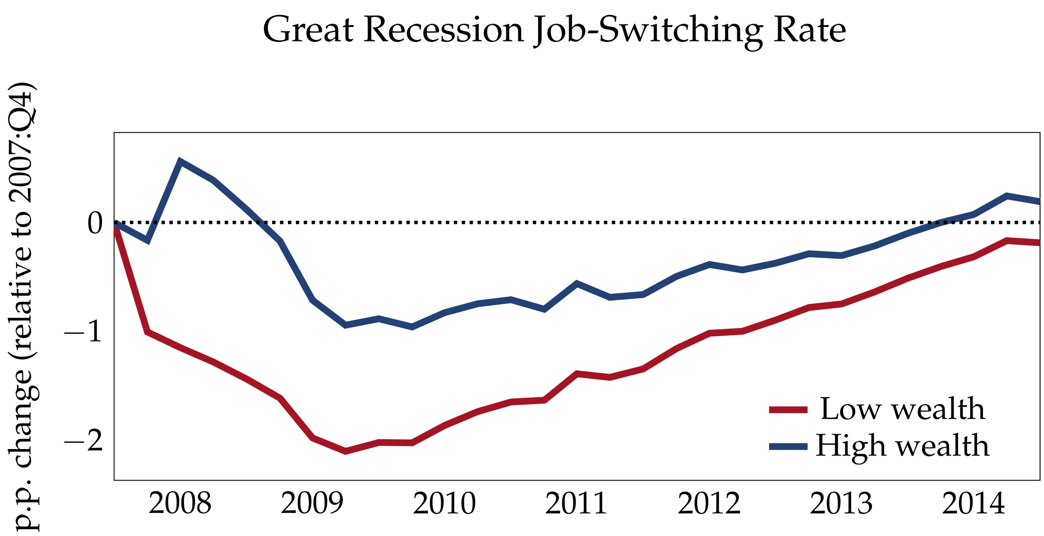


Figure 8: Percentage point change in the job-switching rate implied by the model for low- (red) and high-wealth (blue) workers during the 2007-09 recession.

7.2 Great Reallocation

The US economy behaved very differently after the Pandemic than after the Great Recession. While the model I develop is not tailored to speak to the exceptional economic outcomes of the Pandemic, it sheds light on the unusual job-switching behavior observed during this period. Unlike after the Great Recession, in which the job-switching rate stagnated, the recovery to the Pandemic Recession saw a small fall followed by a fast recovery in job-switching, a behavior so noteworthy it is referred to as the *Great Reallocation*.

One of the aspects that sets the Pandemic apart from previous recessions is the size of the fiscal response. According to the IMF,[20] the three main fiscal stimulus bills passed by Congress, the CARES act, the CAA, and the ARP injected roughly 20% of GDP into the

[20]See https://www.imf.org/en/Topics/imf-and-covid19/Policy-Responses-to-COVID-19.

economy. This stimulus led a growth in household net-worth, especially at the bottom of the distribution (see appendix A for details).

According to the model, higher wealth should have relaxed precautionary job-keeping and increased the willingness of workers to switch jobs. I test my model vis-à-vis the data by subjecting the calibrated model to two shocks to mimic the fiscal response during the pandemic. I subject the economy to transfer shocks, T_t, and unemployment benefits shocks, b_t, lasting six quarters. These shocks match the 3.9% and 2.7% of GDP devoted to direct payments and unemployment support, respectively, in the CARES, CAA, and ARP acts. Figure 9 shows what the model implies for the evolution of the job-switching rate after the Pandemic in various counterfactual scenarios.

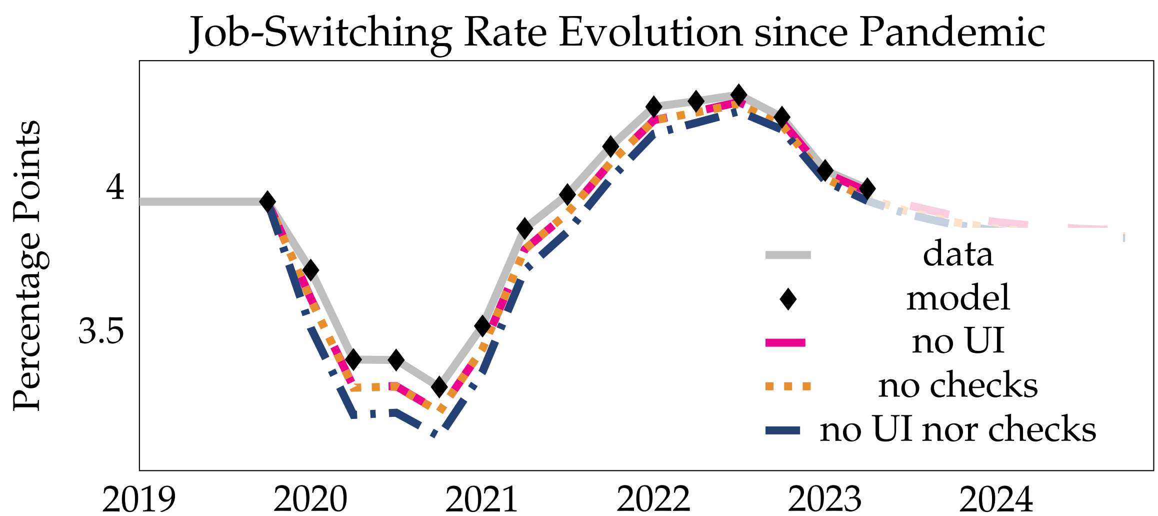


Figure 9: Evolution of the job-switching rate post-Pandemic. The data (and model shocked to mimic the data) are in gray. Three counterfactual scenarios varying the generosity in fiscal transfers are shown in the dashed, dotted, and dash-dotted lines.

The solid gray line shows the evolution of the job-switching rate in the data as well as in the model (black diamonds) subject to the transfer and unemployment benefits shocks discussed above as well as a series of shocks to the vacancy-posting cost κ_t, selected to exactly match the data counterpart.[21] The other lines in figure 9 show what happens by removing the fiscal support packages, one at a time. The dashed magenta line shows the evolution of the job-switching rate absent the additional unemployment benefits, but maintaining direct payments. The dotted orange line shows the flip side of that: agents received additional unemployment benefits but no direct payments. Finally, the blue

[21] Note, because the SIPP does not reach this far I use the change in the job-to-job transition rate estimated in Fujita, Moscarini and Postel-Vinay (2020), at a quarterly frequency, starting from 2019:Q4.

dash-dotted line shows the job-switching rate absent all stimulus to households.

Figure 9 shows that government stimulus boosted job-switching following the Pandemic. The fiscal injection put money in worker's pockets, alleviating their precautionary job-keeping motive and sustaining the recovery in job-to-job transitions. Absent the stimulus, the job-switching rate would have fallen by an extra 20 basis points at its trough.

8 Conclusion

In this paper I ask why workers with different wealth experience such different recoveries in their earnings following economic downturns. This earnings gap, which I document was particularly large in the Great Recession, implies that low-wealth workers, who are worse equipped to confront downturns, are also those hit hardest by them.

To answer this question, I build a general equilibrium DMP model with on-the-job search and incomplete markets. A key ingredient of the model is risky job-switching: workers who switch jobs experience a persistent increase in the risk of subsequent job-loss. Risky job moves arise because tenure determines workers' layoff probability and so workers who switch jobs, moving from higher to lower tenure, experience an increase in their job-loss probability. I quantify this additional job-loss probability to be economically large, roughly a 7 percentage point increase in the probability of job-loss over the first five quarters at a new job. To solve the model I develop a generalized alternating offer bargaining protocol that accommodate on-the-job search, risk-aversion, and asset accumulation. The model delivers two forces linking workers' job-switching and job-losing behavior to wealth. Wealth is tied to job-switching through the phenomenon I denote *precautionary job-keeping* and to job-losing through the *tenure-wealth correlation*.

These mechanisms allow the model to make sense of the cyclical distributional variation in the job-switching and job-losing probabilities across the wealth distribution and in turn help the model explain roughly half of the earnings gap experienced after the Great Recession between low- and high-wealth workers. In addition, the model provides a rationalization of the Great Reallocation, the strong recovery in job-switching observed following the Pandemic. The model implies that the generous fiscal stimulus that accompanied the recession sustained job-switching and facilitated the Great Reallocation.

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A Appendix

A.1 Data

All data used for the paper is publicly available. The empirics (with two exception) are done using SIPP. The first exception is the use of data from Fujita, Moscarini and Postel-Vinay (2020) when studying the aggregate job-switching rate in the Pandemic period. This is because SIPP is not available at such a late date. The second exception is the use of the PSID to compute wealth quintiles as targeted moments in the calibration.

A.1.1 SIPP

I closely follow Nagypál (2008) when constructing labor market variables. I use data from the 1996 survey through the 2008 survey inclusive. I exclude data before 1996 because a significant redesign occurred in that year leading to a larger, less restrictive, and higher quality sample as well as a longer panel with better sampling practices. I exclude data post-2014 because the SIPP went through a redesign then aimed at reducing costs. This led to annual rather than quarterly surveying respondents. All my analysis is done on either the head of the household or their spouse.

Labor Variables in SIPP. As Nagypál (2008), I categorize workers' employment status and workers' firm identifier based on what they report for the last week of each month. A small share of workers have multiple jobs. I restrict attention to their "main" job which is the job they worked the most hours at. The earnings I consider throughout the paper, as well as all job-specific variables, are those for this "main job". Employed workers are those indicated so by their labor status who are working full-time (at least 35 hours per week). I exclude the self-employed, I do so because other forms of risk, such as credit risk, may be more important than the risk of job loss.

Using employment status of workers and the firm ID they are attached to, I construct labor flows. Movements from employment to unemployment in consecutive months are recorded as EU transitions; movements from unemployment to employment as UE transitions. When a worker is employed in two consecutive months (i.e. with no intermediate unemployment spell), they see a change in their firm identifier, and this ID is not listed in the workers' history of firm IDs, then I record such an event as an EE transition. I exclude recall to previous employment because the job-switches I have in mind are ones in which the quality of the firm-worker match is unknown; when workers go back to firms they previously worked for, the quality of the firm-worker match is known.

Wealth in SIPP. The benchmark wealth measure is net-worth. To construct this I follow Kaplan and Violante (2014). The components that go into the asset side are: (i) savings and checkings, (ii) US savings bonds, (iii) Equity in investments, (iv) value of 401K and IRA, (v) value of interest-earnings accounts, (vi) value of stocks and funds, (vii) business equity, (viii) value of vehicles, and (ix) property value. To these I detract liabilities corresponding to: (i) credit-card and store bills debt, (ii) amount owed for loans, (iii) debt on stocks/funds, (iv) vehicle debt, (v) business debt, (vi) other debt, and (vii) principal owed on property. All dollar values (for wealth but also earnings) are deflated using the CPI and are in 2010USD.

A.1.2 Residualized Moments

The job-flow moments residualized by a polynomial in age, sex, race, marital status, industry and education fixed effects are shown in table 5, respectively.

	Stdv.			Half-life		
	all	low-wealth	high-wealth	all	low-wealth	high-wealth
UE	3.38 (0.574)	3.12 (0.543)	3.81 (0.658)	0.9632	0.9599	0.9620
EU	0.54 (0.068)	0.69 (0.065)	0.40 (0.046)	0.8872	0.8838	0.8845
EE	3.89 (0.616)	4.95 (1.09)	3.57 (0.447)	0.0.8712	0.8892	0.8564

Table 5: Quarterly labor market flow rates residualized by polynomial in age, race, sex, industry FE, and education FE. "All" is entire sample, "low wealth" and "high wealth" are the bottom and top halves of the net worth. Standard deviations and half-lives computed on the Hamilton-filtered rates. All data are computed using SIPP 1996-2013.

A.1.3 Wealth Evolution During Past Recessions

Figure 10 compares the evolution of net-worth across wealth. Unlike the 2001 and 2007-09 recessions, the Pandemic recession saw a rapid rise in wealth especially for low-wealth workers. This was in large part due to the sizable fiscal stimulus.

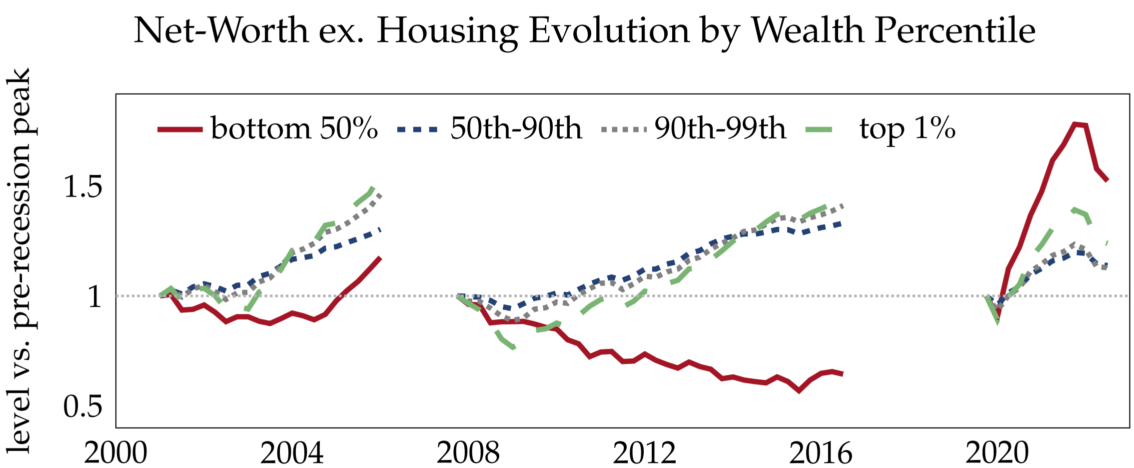

Figure 10: Evolution of net worth from pre-recession peak by wealth percentiles. Source: Distributional Financial Accounts, Board of the Federal Reserve System.

A.2 Bargaining with Employed Agents

Proof. (Result 1)
Consider the wage at m making the incumbent firm indifferent between opening a vacancy and signing on the worker. This indifference is:

$$\left(\epsilon p_n\left[Zf(k)-r^K\kappa\right]-w^*\right)+\beta\mathbb{E}\left[J\left(\psi_a,\epsilon',w^*,n,j+1\right)\right]=0$$

It is clear that there is no dependence on m and thus the solution $\overline{w}_{m,n}$ will also be independent of m. That is $\overline{w}_n := \overline{w}_{1,n} = \ldots \overline{w}_{M,n}$ and $\overline{w}_{m',n}$

Consider the indifference condition for poaching firm n':

$$\frac{M-m+1}{M}\left(\epsilon p_{n'}\left[Zf(k)-r^K\kappa\right]-w^*\right)=-\frac{1}{1+r}\mathbb{E}\left[J\left(\psi_a,\epsilon',w^*,n',0\right)\right]$$

While the RHS is constant in m, the LHS shifts down as m increases and hence $\overline{w}_{m,n'} > \overline{w}_{m',n'}$ for $m' > m$ as shown in the figure below.

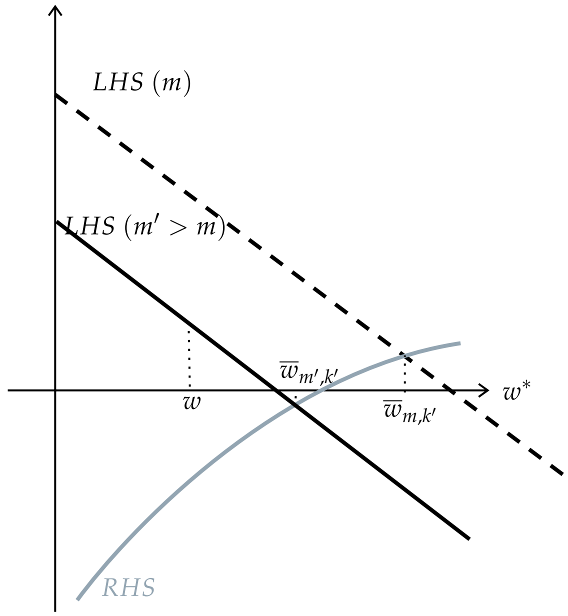

Figure 11: Lemma 1 graphic proofs.

□

Proof. (Result 2)
From Lemma 1 and the terminal values for the worker defined in 21, it follows immediately that $W_1^n(\overline{w}_{1,n}) = \ldots W_M^n(\overline{w}_{M,n})$. From Result 1 and the terminal values for the worker defined in 23, it follows immediately that $W_1^{n'}(\overline{w}_{1,n'}) > \cdots > W_M^{n'}(\overline{w}_{M,n'})$. □

B Online Appendix

B.1 Model

Here I consider the model blocks and the pertaining equations, as well as some useful model-related notes.

B.1.1 Job Market Flows with On-the-Job Search

Because the model is in discrete time, the job-finding and vacancy-filling rates are probabilities rather than arrival rates. Here I define appropriate boundaries for the labor market tightness parameters so that neither the job-finding nor the job-posting probabilities are outside the interval $[0, 1]$. The following inequalities must hold:

$$0 \leq q(\theta_1) \leq 1 \quad \Longleftrightarrow \quad 0 \leq \chi \left(\frac{1}{\theta_1}\right)^{\eta} \leq 1$$

$$
\begin{array}{rcl}
& \Rightarrow & \chi^{\frac{1}{\eta}} \leq \theta_1 < \infty \\
0 \leq \lambda\left(\theta_1\right) \leq 1 & \Longleftrightarrow & 0 \leq \chi \theta_1 \left(\frac{1}{\theta_1}\right)^{\eta} \leq 1 \\
& \Rightarrow & 0 \leq \theta_1 < \chi^{\frac{1}{\eta-1}}
\end{array}
$$

Since $\chi < 1$, it must be that

$$
\theta_1 \in [\chi^{\frac{1}{\eta}}, \chi^{\frac{1}{\eta-1}}] \tag{A.1}
$$

B.1.2 Walras' Law

I show that Walras' law holds. First aggregating the budget constraint of unemployed workers gives

$$
\begin{aligned}
\int_i c_t(i) + a_{t+1}(i)\, di &= \int (1+r) a_t(i) + (1-\tau_t) b + T_t\, di \\
& \quad \text{where the integral is over unemployed agents} \\
C_t^U + A_{t+1}^U &= (1+r) A_t^U + ((1-\tau_t) b + T_t) \int di^U
\end{aligned}
$$

Doing the same exercise for the employed gives

$$
C_t^E + A_{t+1}^E = (1+r) A_t^E + (1-\tau_t) \int w(i^E)\, di^E + T_t \cdot \left(1 - \int di^E\right)
$$

Summing the two gives

$$
\begin{aligned}
\left(C_t^U + C_t^E\right) + \left(A_{t+1}^U + A_{t+1}^E\right) &= (1+r_t)\left(A_t^U + A_t^E\right) + T_t \\
& \quad + (1-\tau_t) b \cdot \int di^U + (1-\tau_t) \int w(i^E)\, di^E \\
C_t + A_{t+1} &= (1+r_t) A_t + \int w(i^E)\, di^E \\
& \quad + T_t + (1-\tau_t) b \int di^U - \tau_t \int w(i^E)\, di^E \\
& \quad \text{using the balanced budget equation} \\
C_t + A_{t+1} &= (1+r_t) A_t + \int w(i^E)\, di^E + \Pi_t
\end{aligned}
$$

Recall also that the flow of profits of an individual firm is simply

$$
\pi = (p_n \cdot \epsilon)\left[f(k) - r^K k\right] - w
$$

which, aggregating over all firms and netting out the total vacancy costs gives

$$\Pi_t = \int \left(p_n \cdot \epsilon(i^E)\right) \left[f\left(k\right) - r^K k\right] - w(i^E)\ di^E - \kappa \cdot \sum_{n=1}^{N} v_n = Y_t - \int \left(p_n \cdot \epsilon(i^E)\right) r^K k + w(i^E)\ di^E - \kappa \cdot \sum_{n=1}^{N} v_n$$

Including this in the equation above gives

$$\begin{aligned} C_t + A_{t+1} &= (1+r_t)A_t + \int w(i^E)\ di^E + Y_t - \int \left(p_n \cdot \epsilon(i^E)\right) r^K k + w(i^E)\ di^E - \kappa \cdot \sum_{n=1}^{N} v_n \\ C_t &= Y_t + ((1+r_t)A_t - A_{t+1}) - r^K K_t - \kappa \cdot \sum_{n=1}^{N} v_n \end{aligned}$$

Note, asset market clearing implies that $p_t\left(K_t\right) = A_t$ and $p_{t+1}\left(K_{t+1}\right) = A_{t+1}$, and thus we can rewrite this expression as

$$\begin{aligned} C_t &= Y_t + \underbrace{(1+r_t)\,p_t\left(K_t\right) - p_{t+1}\left(K_{t+1}\right)}_{D_t} - r^K K_t - \kappa \cdot \sum_{n=1}^{N} v_n \\ &= Y_t + \underbrace{(1+r_t)\,p_t\left(K_t\right) - p_{t+1}\left(K_{t+1}\right)}_{D_t} - r^K K_t - \kappa \cdot \sum_{n=1}^{N} v_n \end{aligned}$$

Substituting the definition of dividends from 11 gives

$$\begin{aligned} C_t &= Y_t + r_t^K K_t - \left[K_{t+1} - (1-\delta)\,K_t + \frac{1}{2\delta\epsilon_I}\left(\frac{K_{t+1}-K_t}{K_t}\right)^2 K_t\right] - r^K K_t - \kappa \cdot \sum_{n=1}^{N} v_n \\ C_t &= Y_t - \left[K_{t+1} - (1-\delta)\,K_t + \frac{1}{2\delta\epsilon_I}\left(\frac{K_{t+1}-K_t}{K_t}\right)^2 K_t\right] - \kappa \cdot \sum_{n=1}^{N} v_n \qquad \text{(A.2)} \end{aligned}$$

that is, consumption equals output minus investment and capital adjustment costs. In steady state this boils down to the economy-wide resource constraint

$$C = Y - \delta K - \kappa \cdot \sum_{n=1}^{N} v_n$$

Labor flow rates. Vacancies opened today result in matches tomorrow. The job-finding and vacancy-filling rates are then

$$q_{t,n} = \chi\left(\frac{1}{\theta_{t-1,n}}\right)^{\eta} \qquad \text{(A.3)}$$

$$\lambda_{t,n} = \theta_{t-1,n} \cdot q_{t,n} \tag{A.4}$$

Matching Technology. To compute the vacancies posted by firms at time t given the mass of searchers on each rung, $e_{t,n}$

$$v_{t,n} = \theta_{t,n} \cdot e_{t,n} \tag{A.5}$$

Labor + Investment (solved). For labor the standard CRS equation holds:

$$L_t = \left(\frac{Y_t}{Z_t K_{t-1}^{\alpha}}\right)^{\frac{1}{1-\alpha}} \tag{A.6}$$

$$r_t^K = \alpha Z_t \left(\frac{L_t}{K_{t-1}}\right)^{1-\alpha} \tag{A.7}$$

and for investment

$$Q_t = \frac{1}{\delta \epsilon_I}\left(\frac{K_t}{K_{t-1}} - 1\right) + 1 \tag{A.8}$$

$$Q_t = \frac{1}{1+r_t}\mathbb{E}\left[r_{t+1}^K - \frac{K_{t+1}}{K_t} + (1-\delta) - \frac{1}{2\delta\epsilon_I}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 + \frac{K_{t+1}}{K_t}Q_{t+1}\right] \tag{A.9}$$

Dividend. The dividend spits out investment and the dividend

$$\phi(K_t, K_{t-1}) = K_{t-1} \cdot \frac{1}{2\delta\epsilon_I}\left(\frac{K_t}{K_{t-1}} - 1\right)^2 \tag{A.10}$$

$$I_t = K_t - (1-\delta)K_{t-1} + \phi(K_t, K_{t-1}) \tag{A.11}$$

$$D_t = r_t^K K_{t-1} - I_t \tag{A.12}$$

Capitalist. The dividend is priced such that the (ex-ante) real interest rate is

$$1 + r_t = \frac{\mathbb{E}\left[p_{t+1} + D_{t+1}\right]}{p_t} \tag{A.13}$$

Intermediaries. The intermediary blocks sets the deposit rate (that HH take)

$$1 + r_t^a = \frac{D_t + p_t}{p_{t-1}} \tag{A.14}$$

Objectives. And the objective functions are

$$A = p \tag{A.15}$$

$$N = L \tag{A.16}$$

$$Tax_t + \Pi_t - \zeta\left(\sum_{k=0}^{K} v_{t,n} p(k)\right) = (1-\tau_t) UI_t + T_t \tag{A.17}$$

B.1.3 Micro-founding the Job-Loss Probability

The job-loss probability that is decreasing in tenure can be microfounded differently from the way I do in the body of the paper. Here I including learning on the job about the quality of the match between worker and firm in the spirit of Jovanovic (1979).

Setup. The quality of the firm-worker match $\overline{y}$ is not observable, rather the firm observes a noisy version of it

$$y = \overline{y} \cdot \omega \tag{A.18}$$

where ω is noise. The true match-quality can be high, y_H, or low, y_L, with unconditional probabilities π^0 and $1-\pi^0$, respectively.

For simplicity, assume the true match quality is revealed at $j = J$, that is, after the match persists long-enough, there is no more uncertainty about its quality. Before it, however, ω is distributed according to a mean zero probability mass function $h(\omega)$ with support $[\underline{\omega}, \overline{\omega}]$ (where $H(\omega)$ is the cdf).

Separation Rate. The match is dissolved whenever the firm prefers its outside option to sticking to the worker. I consider here a worker who previously agreed on a wage w with the firm. Worker and firm values depend on the other states of the problem (e.g. assets) but I suppress them in the following notation for convenience and simply write the firm's value as a function of output and the wage paid, $J(\overline{y}, w)$.

If the realized output y is very low, the firm will think the worker is of low quality and will opt to terminate the relationship. Thus, if the firm observes y from the worker, it will update its prior on the worker being high quality according to Bayes' rule. If π_{j-1} is the probability of the match being of high quality at tenure $j-1$, the probability at j is

$$\pi_j = \frac{\pi_{j-1} Pr(\omega = y^H - y_j)}{\pi_{j-1} Pr(\omega = y^H - y_j) + (1-\pi_{j-1})\, Pr(\omega = y^L - y_j)}$$

where y_j is the output observed at j. The value the firm expects to extract from the worker is

$$J^{\text{keep}}\left(y_j,w\right)=\left(y_j-w\right)+\frac{1}{1+r}\left[\pi_1 J\left(y^H,w\right)+(1-\pi_1)J\left(y^L,w\right)\right]$$

while the value the firm would get by laying off the worker is

$$J^{\text{fire}}\left(y_j,w\right)=\left(y_j-w\right)+\frac{1}{1+r}V=\left(y_j-w\right)$$

where V is the value of vacancy. The firm will then fire the worker whenever $J^{\text{fire}}>J^{\text{keep}}$.

B.2 Bargaining: Equilibrium Strategies

There is complete information and agents know each others' valuations as well as all relevant *waiting options*, that is the values agents receives at $m+1$ if no agreement is reached at m. In what follows, the guiding principle is the standard logic of alternating offer games: agents make the lowest offer that allows them to sign the contract (conditional on this not leading to a lower value than their outside option).

When m is odd, the firms bid for the worker. The valuations they have for the worker are $\overline{W}^{n}$ for the incumbent n (recall there is no dependence on m) and $\overline{W}^{n'}_{m}$ for the poacher n' (recall this is decreasing in m). Denote the worker's outside option, i.e. the value they receive at $m+1$, as W^{wait}_{m+1}. This is $W^n(w)$ if $m=M$ as the bargaining breaks down and the worker returns to firm n at the original wage w. Four possibilities arise:

1) If $\overline{W}^{n}>W^{\text{wait}}_{m+1}\geq\overline{W}^{n'}_{m}$, the maximal offer firm n' can make does not compete with the value the worker receives in the following sub-period. The relevant outside option is therefore W^{wait}_{m+1} because the worker can always wait until the next period and earn that value. Firm n' will offer the best it can, $\overline{W}^{n'}_{m}$, but still have no hope of poaching the worker, and firm n will offer the minimum value to retain the worker, that is one penny more than $\max\{W^{\text{wait}}_{m+1},\overline{W}^{n'}_{m}\}=W^{\text{wait}}_{m+1}$. The worker accepts the offer firm n makes and is retained by firm n at the wage w^* satisfying $W^n(w^*)=W^{\text{wait}}_{m+1}$. What does this case correspond to? To answer this we must know what W^{wait}_{m+1} is. Note, this outside option dominates $\overline{W}^{n'}_{m}$ which, by result (2), is decreasing in m. This means W^{wait}_{m+1} cannot be determined by the poacher. The only possibility is for the outside option to be $W^{n}\left(w\right)$. So, what this case implies is that the worker remains at the incumbent firm at the original wage w.

2) If $\overline{W}^{n}\geq\overline{W}^{n'}_{m}>W^{\text{wait}}_{m+1}$, $\overline{W}^{n'}_{m}$ is the relevant outside option for the worker. This is because firm n' is driven to offer the worker $\overline{W}^{n'}_{m}$ otherwise it cannot poach the worker.

Firm n matches that offer (and offers an infinitesimal more in value) to retain the worker at wage w^* satisfying $W^n(w^*) = \overline{W}_m^{n'}$. This corresponds to Bertrand competition where the winning firm, the incumbent, offers the worker the maximum value the poacher can deliver.

3) If $\overline{W}_m^{n'} > \overline{W}^n > W_{m+1}^{\text{wait}}$, $\overline{W}^n$ is the relevant outside option for the worker. Firm n offers all it can, $\overline{W}^n$. Firm n' matches that offer (and offers an infinitesimal more in value) to poach the worker at wage w^* satisfying $W_m^{n'}(w^*) = \overline{W}^n$. This too corresponds to Bertrand competition where the winning firm, the poacher, offers the worker the maximum value the incumbent can deliver.

4) If $\overline{W}_m^{n'} \geq W_{m+1}^{\text{wait}} > \overline{W}^n$, W_{m+1}^{wait} is the relevant outside option of the worker. It is important to note that this case can only occur if, by waiting until $m+1$, the worker would still prefer the poacher since W_{m+1}^{wait} could not have been derived from the incumbent whose break-even value (constant in m) $\overline{W}^n$ is lower than this waiting value. Firm n offers all it can, $\overline{W}^n$ but has no hope of retaining the worker. Firm n' offers the bare minimum in order to hire the worker, that is $\max\{W_{m+1}^{\text{wait}}, \overline{W}^n\} = W_{m+1}^{\text{wait}}$. n' poaches the worker at wage w^* satisfying $W_m^{n'}(w^*) = W_{m+1}^{\text{wait}}$. What does this case correspond to? Because W_{m+1}^{wait} is pinned down by the poacher, here we have the poacher competing against time (future offers it is able to make), this is the one-on-one AOB in action.

When m is even, the worker starts by making an offer to firm n'. It proposes a wage that makes n' indifferent between accepting the offer and waiting until the next sub-period. The following cases arise:

1) Suppose $\overline{W}^n > W_{m+1}^{\text{wait}} \geq \overline{W}_m^{n'}$. If the worker were to make an offer to firm n' it would make it indifferent between accepting the wage and waiting until sub-period $m+1$. Result (2) implies that $\overline{W}^n > W_{m+1}^{\text{wait}} \geq \overline{W}_m^{n'} > \overline{W}_{m+1}^{n'}$. The strategies at $m+1$ imply firm n' will not be able to poach the worker in any of the next sub-periods and will in fact remain vacant. This means the worker is able to extract all the match value $\overline{W}_m^{n'}$ from the poacher. The worker asks firm n to match the best available offer which is $\max\{W_{m+1}^{\text{wait}}, \overline{W}_m^{n'}\} = W_{m+1}^{\text{wait}}$. Thus, firm n retains the worker at wage w^* satisfying $W^n(w^*) = W_{m+1}^{\text{wait}}$. It is worth pointing out that, just as in case 1) with m odd, W_{m+1}^{wait} must equal $W(w)$. Thus, the worker stays with the incumbent n at the original wage w.

2) Suppose $\overline{W}^n \geq \overline{W}_m^{n'} > W_{m+1}^{\text{wait}}$. Just as before, if the worker made an offer to the poacher, it would be able to extract the entirety of the value $\overline{W}_m^{n'}$ since the poacher

knows the incumbent can beat its offer. The worker then asks firm n to match the best outside offer the worker has, that is $\max\{W^{\text{wait}}_{m+1}, \overline{W}^{n'}_{m}\} = \overline{W}^{n'}_{m}$. Firm n will match $\overline{W}^{n'}_{m}$ in order to avoid having firm n' poach the worker. The new wage firm n and the worker agree on is w^* satisfying $W^n(w^*) = \overline{W}^{n'}_{m}$.

3) Suppose $\overline{W}^{n'}_{m} > \overline{W}^{n} \geq W^{\text{wait}}_{m+1} \geq \overline{W}^{n'}_{m+1}$. At $m+1$ firm n is able to provide the worker more value than firm n'.[22] Thus, at m, the outside option of firm n' is to post a vacancy. The worker, making the poacher indifferent with its outside option, extracts all the value from firm n' and obtains value $\overline{W}^{n'}_{m}$. Firm n will fail to match this offer. The worker is poached by firm n' at wage $\overline{w}^{n'}_{m}$.

4) Suppose $\overline{W}^{n'}_{m} > \overline{W}^{n'}_{m+1} > W^{\text{wait}}_{m+1} \geq \overline{W}^{n}$, then at $m+1$ firm n' would still retain the worker as per the strategies described above[23]. The worker then offers a wage w^*_m making the firm indifferent between accepting and moving on to the next sub-period, that is $\frac{M-m+1}{M} y_{n'} + \frac{1}{1+r}\mathbb{E}\left[J^{n'}(w^*_m)\right] = J^{n'}_m(w^*_{m+1})$.

Note that in case 4) firm n is irrelevant. How then is the outside option of firm n' determined? The worker and firm n' alternate making offers that make the other party indifferent between accepting and waiting until the next sub-period, until, at some m^{end} firm n' cannot count on retaining the worker at m^*+1. This occurs when $W^{n'}_{m}\left(\overline{w}^{n'}_{m^{\text{end}}}\right) > \overline{W}^{n} \geq W^{\text{wait}}_{m+1} \geq W^{n'}_{m^{\text{end}}+1}\left(\overline{w}^{n'}_{m^{\text{end}}+1}\right)$. At this m^{end} the strategy is exactly as described in case 3) for both m odd or even. Firm n' poaches the worker but must match the total value n can offer the worker. In the previous sub-periods $m < m^*$ firm n' and the worker negotiate bilaterally knowing what will happen at m^{end}.[24]

[22] Again, W^{wait}_{m+1} could not have come from the poacher at $m+1$ since it dominates the break-even valuation of the poacher then.

[23] It can be shown that $W^{\text{wait}}_{m+1} \geq \overline{W}^{n}$, otherwise firm n would be able to offer $\overline{w}^{n}$ to the worker and dominate the outside offer which would in itself imply W^{wait}_{m+1} is not the outside offer.

[24] If no such m^{end} exists, $m^{\text{end}} = M$ and the firm offers the worker exactly what firm n is able to offer.